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SPECTROPHOTOMETRIC DETERMINATIONS OF CONTACT AT TOTAL ECLIPSES OF THE SUN

BY

HENRIK KRISTENSON

WITH 28 FIGURES IN THE TEXT

COMMUNICATED 24 JANUARY 1951 BY BERTIL LINDBLAD AND YNGVE ÖHMAN

STOCKHOLM

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LONDON
H. K. LEWIS & CO., LTD
136, GOWER STREET

PARIS
LIBRAIRIE C. KLINCKSIECK
11 RUE DE LILLE

1951

SPECTROPHOTOMETRIC
DETERMINATIONS OF CONTACT AT
TOTAL ECLIPSES OF THE SUN

INAUGURAL DISSERTATION

BY

HENRIK KRISTENSON

FIL. LIC., HLD.

BY DUE PERMISSION OF THE PHILOSOPHICAL
FACULTY OF THE UNIVERSITY OF LUND, TO
BE PUBLICLY DISCUSSED AT THE PHYSICAL
INSTITUTE MAY 19TH, 1951, AT 10 A. M., FOR THE
DEGREE OF DOCTOR OF PHILOSOPHY

UPPSALA 1951

ALMQVIST & WIKSELLS BOKTRYCKERI AB



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With 28 figures in the text

Communicated 24 January 1951 by BERTIL LINDBLAD and YNGVE ÖRMAN

Preface

The present investigation which deals mainly with the timing of a total solar eclipse by means of a procedure suggested by Professor B. LINDBLAD in 1943 started at the Lund Observatory on the basis of material secured at the solar eclipse on July 9, 1945. I am respectfully indebted to Professor K. LUNDMARK, Director of the Lund Observatory, for generously putting at my disposal the cinematographic recording obtained at this eclipse. I thank Professor LUNDMARK also for his encouraging support and valuable advice during my entire time at the Lund Observatory.

The investigation has been extended and completed during a two years stay at the Stockholm Observatory where, besides work in other fields, the present subject could be considered in most profitable discussions with Professor B. LINDBLAD, Director of the Stockholm Observatory. The experiences and results gained by him at the treatment of the photometric material of his own expedition at the same eclipse were generously imparted to me so that the investigation herewith presented includes the contact results of the Stockholm Observatory expedition as well. The empirical basis of the actual procedure for determination of contacts could in this way be considerably widened. For this and for all interest shown in my work at the Stockholm Observatory it is a pleasant duty to express to Professor LINDBLAD my deep and respectful gratitude.

Stockholm Observatory, Saltsjöbaden, Jan. 1951.

HENRIK KRISTENSON

CHAPTER I

Introduction to the problem of contact determinations with special regard to geodetic applications

1. *General.* The idea of using total or annular solar eclipses for the purpose of determining the size and the shape of the Earth seems to be rather old. The underlying principle is, to be sure, very simple: the distance between two observers watching the eclipse from two separate points on the central line can be derived from the time interval during which the shadow cone sweeps from one observer to the other and — formally speaking — from the speed of the shadow relative to the surface of the Earth. An investigation of this kind must accordingly be carried out in two steps, one observational consisting in establishing the times of contact at each station and one computational including an accurate study of the motions of the Earth and the Moon during the time involved. When from this procedure there is required an accuracy in the determination of distance approaching that of ordinary triangulation, the following two conditions must be fulfilled:

(a) The theories of the solar and lunar motion must be brought up to a very high level to be able to yield, with sufficient accuracy, the relative motions taking place between the two observations. As to this point, the present situation is far more favourable than it was, say, a century ago. The careful investigations by S. NEWCOMB and E. W. BROWN, as condensed in their *Tables of the Sun* and *Tables of the Moon* respectively, furnish the astronomer with the requisite data which, moreover, may be improved by computing with great accuracy the instantaneous perturbations exerted on the Earth and the Moon during the critical time interval.

(b) The exactitude of the contact determinations must be increased considerably above that obtained from the classical procedures. These are based, generally, on direct measurements on the uncovered solar crescent, photographed by means of rather big instruments. These photographs have to be distributed over the entire partial phase and accordingly over a considerable interval of time. Now, if we require from the determination of distance an accuracy of, say, 30 m and assume the speed of the shadow (relative to the observers) to be of the order 1000 m per second, we readily find the corresponding permitted error in time to be as small as some few hundredths of a second. Such an accuracy will scarcely be attained on the basis of the classical contact methods. For even if the internal mean errors affecting the contact times would come

out rather small, the observer will, nevertheless, run the risk of having his results suffer from severe systematic errors. These may be caused not only by the irregular and not too well known shape of the lunar edge but also by more or less uncontrollable alterations of the plate properties, the exposure time, the air transparency and by difficulties connected with the personal interpretation of the blurred borders (photographic irradiation!) of the Sun and the Moon. The state of things is made clearer by considering the particular case that we are to utilize a plate, secured by means of an instrument of $f = 2$ m, for the purpose of establishing the relative position between the Sun and Moon. On this plate, the granulae of the photographic emulsion subtend an angle of some $0''.10$ whereas, as we are soon going to see, the realization of the geodetic idea requires an accuracy in the relative position of but some $0''.02$!

2. *Modern principles of contact determination.* The most natural way to overcome the drawbacks of the older methods would be to reject the geometrical treatment of the contact problem and arrange instead the observations in such a manner that the measuring machine may be avoided as a main tool when evaluating the secured material. Measurements of the light emitted from the uncovered solar crescent offer such a mode of procedure and evidently, on this basis, the best results should be expected to come from a study of the abrupt change in light intensity which takes place at the beginning and end of the total phase. Another mode of procedure is to investigate in the immediate neighbourhood of maximum eclipse the swift swing-around in position angle of the solar crescent as observed from a point just outside the belt of totality. The rapid progress of the actual phenomena was formerly a severe obstacle for following them into fine details. The introduction of the cinematographic recording technique has removed this obstacle completely; the same advantage has been achieved in introducing the photoelectric cell into astronomical praxis.

Eclipse timing methods, based on the principles mentioned above, have been proposed and attempted by different astronomers. A survey of these procedures will be given in the following sections, which at the same time aim at giving the chronological background of the present investigation.

3. *The method of BANACHIEWICZ.* The first attempt to time a solar eclipse with a direct aim at a geodetic application was made by T. BANACHIEWICZ [3] in 1936 (June 19). On this occasion, after having gained favourable experiences with regard to the precision of the contact times from his "chrono-cinematographic" recording of a previous eclipse [2] (June 29, 1927), he stationed three expeditions in Greece, Siberia

and Japan with the intention of accurately establishing the moments of contact. An interesting point, amongst others, was the use of special time signals transmitted from Warsaw and intended to furnish the expeditions with a common reliable time scale. It is regrettable only in that, so far, no account has appeared giving the results of this far-reaching attempt.

The chronocinematography of 1927 (and 1936) consisted in a timed high speed registering of the eclipse close to the four contacts achieved by means of a movie camera of focal length 1.20 m. To derive the times of the inner contacts, K. KORDYLEWSKI [16], from a mere inspection by eye of the records, stated the moments when the photospheric light disappeared or reappeared at a great many points along the whole solar crescent. The height corresponding to any given detail of the lunar contour was obtained from the topographic map by F. HAYN [10]. KORDYLEWSKI admits that the moments in question were rather difficult to determine, primarily because of the turbulence in the terrestrial atmosphere, which caused the Baily's beads to behave in a quite undisciplined manner. In spite of this, the difference in right ascension (Moon-Sun) could be established with a mean error of but $^{\circ}04$. The accuracy of the results from the outer contacts was, of course, considerably less.

It is clear that the method of BANACHIEWICZ-KORDYLEWSKI is able to offer several advantages over the older, geometrical methods, which in fact have been replaced by what could be called a semiphotometric procedure even though this is far from being of an objective nature. Two objections may be raised, however, against this method: first, it is not likely to improve our defective knowledge about the actual contacting lunar contour; second, the determination of contact for any given limb detail is achieved in a qualitative rather than a quantitative manner, that is to say, no relation containing the amount of light registered may be established between the successive pictures. These draw-backs are difficult to overcome simply because of the fact that the crescents cannot easily be subjected to an objective photometry.

4. *The proposal of BONSDORFF.* In 1943, I. BONSDORFF [4] called attention to the interesting possibility of using the total solar eclipse of 1945, July 9, — with total belt stretching between the North American and European continents — for the purpose of connecting the geodetic nets of the continents in question. Ordinary triangulation, unable to bridge the Atlantic Ocean, should thus be replaced by triangulation with the aid of the Moon as the third point.

In his paper BONSDORFF delivers a general discussion on the special difficulties

which are connected with the realization of the geodetic project. There he states three sources of error to be most troublesome and gives the following comments about these:

(a) Errors arising from the adopted constants of the Moon's motion. Since the motion of the Moon is involved for but just a few hours, the only constant in need of improvement is the lunar parallax. It is proposed to do this by observing solar eclipses or occultations from two distant points which are connected by ordinary triangulation.

(b) Errors arising from irregularities in the contour of the lunar limb. Although it is judged difficult to overcome these definitely, the influence exerted on the times of contact can be regarded as being considerably reduced by the fact that on the whole, since differential libration will be very small, the same contour will be observed from both stations. General information about the limb topography could, on the other hand, be obtained for instance from photographic records secured during the partial phase. By this procedure the actual error affecting the determination of distance at one single contact would scarcely exceed 30 m (m. e.).

(c) Errors in measurements of the times of contact. For the solution of this problem, BONSDORFF suggests using the method of BANACHIEWICZ, by means of which, apart from the error caused by the lunar contour, he expects to derive the time difference with a mean error of $0^s.028$. It is presumed that all four contacts are observed and the results from the outer contacts have the same weight as those derived from the inner contacts.

Finally, BONSDORFF proposed that Swedish astronomers participate in the carrying out of his plan by making observations to accurately establish times of contact. They really made considerable contributions to the solution of this important problem, even though they, in so doing, generally followed another procedure, that given by B. LINDBLAD.

5. *The method of LINDBLAD.* For the realization of BONSDORFF's project, B. LINDBLAD [19], in 1943, presented an entirely different idea concerning the determination of contacts. He based his proposal upon an attempt, made in collaboration with Mr. O. WIBERG at the eclipse of June 29, 1927, to follow cinematographically the rapid changes which occur in the chromospheric flash spectrum close to the inner contacts [18]. The instrument employed was a moving picture camera of focal length 25 cm, diaphragmed to 1:12 and delivering 15 exposures per second. The attached objective prism furnished a mean dispersion of some 200 Å per mm.

The flash spectrum proved to be subject to so rapid a change that distinct differences could be revealed in two consecutive recordings, even from a mere inspection by eye. Remembering that during the corresponding interval of time the Moon has shifted its position relative to the Sun only $0''.03$, it is evident that at the Sun's very edge spectral intensity turns out to be a very sensitive indicator of the position of the lunar limb. To settle the times of contact, LINDBLAD therefore suggests undertaking an objective photometry in selected regions of the flash spectrum. The spectral recordings of the solar crescent might, moreover, offer two important advantages over the corresponding direct pictures: first, the influence caused by atmospheric turbulence is nearly eliminated; second, the focal length of the registering instrument may be taken considerably shorter.

Finally, LINDBLAD raises criticism against the geometrical conception of the contact problem in pointing out that the idea of a sharp and well defined solar limb does not hold here. Theoretical investigations have revealed, in accordance with the cinematographic registering mentioned, that the decrease of light at the solar limb does not take place quite abruptly but falls off gradually to zero, though very rapidly, within a certain limited region. Now, the mean error in the determination of every one of the four contacts should according to BONSDORFF [4] not exceed $0''.04$ or, converted into angular measure of the solar radius, $0''.02$ (relative velocity assumed to be $\frac{1}{2}'' \text{ sec}^{-1}$). This angle is small, in fact, compared to the angular depth of the region just mentioned, which shows the actual contact problem to be essentially a photometric problem. Moreover, if achieved from a study of the continuous part of the flash spectrum, the determinations of contact will as a by-product yield valuable information about the sharpness of the solar limb in monochromatic light.

The method just outlined will be discussed in detail in the present paper. It will in what follows be termed *the method of LINDBLAD* or, alternatively, *the (spectro)photometric method*.

6. *The method of PLATZECK, MAIZTEGUI and GAVIOLA* [26]. Following the suggestion of BONSDORFF, members of the Córdoba Observatory tried the method of BANACHIEWICZ to time at Soto, Argentina, the total solar eclipse of May 20, 1947. The attempt was quite unsuccessful because of a completely clouded sky. It prompted, however, the authors listed in the heading, who were struck by the sudden drop of light at the beginning of the total phase, to propose studying the rapid change of the integrated solar light for the purpose of determining the moments of inner contact. Such measurements which

can be arranged photoelectrically or photographically would enable the observer to increase considerably the probability of success in that, even in the case of a covered sky, useful results might be obtained in exposing the light sensitive medium by a large solid angle of light. The influences on the registered light curve which arise from the irregularities of the lunar profile may be accounted for when the shape of the actual contour is approximately known.

The procedure outlined and the method of LINDBLAD are accordingly both based upon photometric measurements but bear, in other respects, not very much resemblance to each other. We shall return to some further considerations concerning this method in the last chapter of the present paper. So far, the procedure in question has not been attempted at any eclipse (cf. however [27]).

7. *The method of ATKINSON.* Another procedure of determining the relative position between Sun and Moon has been tried by R. D'E. ATKINSON [36]. When making the cinematographic registration from a point outside but rather close to the total belt one records instead of a disappearing or reappearing solar crescent a crescent turning round very quickly in position angle. The moment of maximum eclipse distinguished by maximum speed of swing-round may accordingly be settled from a study of the crescent's orientation on the basis of a timed cinematography of the eclipse.

The procedure in question was attempted at the eclipse of November 1, 1948, in Mombasa, Africa; the results are so far unpublished.

8. *The attempts made with the eclipse method up to the present year.* Actually up to 1950, the eclipse method of obtaining geodetic information has been tried on the following occasions:

(a) On *June 19, 1936*, [3], three Polish expeditions, with sites in Greece, Siberia and Japan, observed the total eclipse with special regard to the times of contact. The procedure used was the chronocinematographic method of BANACHIEWICZ (cf. p. 5). The timing of the exposures was achieved in registering directly, on the film containing the exposures, the flash lights from a neon lamp lit at the very moment of the ticks of a chronometer. The chronometers were checked for accuracy by means of special long wave time signals transmitted from Warsaw [13].

(b) On *July 9, 1945*, Finnish and Swedish expeditions, at sites in their own countries, following the suggestion of BONSDORFF, tried to time the eclipse according to the methods of BANACHIEWICZ and LINDBLAD. The original intention of BONSDORFF which was to have a great number of expeditions studying the eclipse from stations in Canada,

Greenland, Scandinavia, Finland and European Russia, was impossible to realize owing to the prevailing political conditions. Accordingly, no measurements of distance between points on the two continents could be arrived at on this occasion. A new opportunity of the same kind will, however, be offered by the total solar eclipse of June 30, 1954, which certainly will be utilized very intensely for the purpose. Predictional data for Scandinavia has already been worked out by the late Dr. H. O. GRÖNSTRAND, of Stockholm Observatory, [8]. — The Swedish contribution to the solution of the contact problem is the theme of the present paper.

(c) On *May 20, 1947*, Finnish [37] and Swedish [38] expeditions in South America and West Africa carried out contact observations aiming at a geodetic connection of these two continents. The Finnish expeditions, stationed in Brazil and at the Gold Coast of Africa, succeeded in securing direct cinematographic recordings at both stations. The Swedish expeditions, with sites in Brazil and French Togoland, employed the spectrophotometric method. The attempt was however less successful since the weather conditions at the observation site in Brazil were quite unfavourable, so that here no registration at all could be secured. — It may be mentioned that the Swedish astronomers, in order to establish the time difference between the contacts obtained at the two stations, made use of not only the ordinary time signals but also of an arrangement based upon a direct comparison of the astronomical clocks by exchanging wireless signals between the two expeditions. The exact timing of the exposures was achieved in different ways by the Finnish and Swedish expeditions. Whereas the former, this time as well as in 1945, recorded the chronometer contacts and the time signals by means of a technique similar to that developed in the production of sound film, the latter employed instead an oscillographic time registration which, in fact, constituted one of the two procedures tried by Swedish astronomers in 1945.

(d) On *May 8—9, 1948* [15], North American expeditions — sponsored by The National Geographic Society — tried a direct cinematography of the annular eclipse from sites in Burma, Thailand, China, Korea, Japan and the Aleutians. Of these expeditions, those in Burma, Thailand and Japan were able to secure, more or less, the intended registrations whence at any rate Japan and southeastern India might be brought into a more reliable geodetic connection than before. In addition to these observations, cinematographic records were obtained from two aircraft of the B-29 type, flying above the clouds at an altitude of 29,000 feet in the region of the Aleutians. The position of these planes could be determined by exchanging radar signals with two ground stations. — The instruments and the procedure of timing the exposures seem to have been similar

to those used by the Finnish geodesists at their attempts in 1945 and 1947. A preliminary report concerning the treatment of the recorded films has been published by T. N. PANAY [25].

CHAPTER II

The total solar eclipse of 1945 July 9 The eclipse expeditions

9. *Characteristics of the eclipse.* The general characteristics of the total solar eclipse of July 9, 1945, were as follows: the umbra striking the Earth for the first time in the state of Idaho, USA, running from there over Canada, Greenland, North Scandinavia, Finland and Russia and, finally, touching the Earth's surface for the last time in the Kirghiz Soviet Republic.

A detailed study of the eclipse characteristics for northwestern Europe was worked out by Dr. H. O. GRÖNSTRAND, in 1944. His computations were published in two separate volumes, giving the prediction data for the Scandinavian peninsula [6] and for Finland [7] respectively.

The late Professor K. F. SUNDMAN [31], on the suggestion of BONSDORFF, carried out a special computation for the purpose of finding, with an accuracy of .01 second of arc, the relative position of the Sun and the Moon during the eclipse as a whole. SUNDMAN did this in starting directly from the differential equations of the motions of the Earth and the Moon and in taking into consideration the disturbances exerted by the planets and the flattening of the Earth. The integration constants were determined by anchoring the computed orbit at its beginning and end to the orbit given by BROWN'S *Tables of the Moon*.

Fig. 1, a result from the predictional work by GRÖNSTRAND, shows the path of the total belt across the northern parts of Norway and Sweden. The indicated isochrones of maximum phase are given in GMT for every other minute of time. The central line duration of totality was found to range from 69 seconds on the western coast of Norway to 65 seconds on the Bothnian coast of Sweden; the corresponding data for the Sun's altitude ranged from $38^{\circ}8$ to $36^{\circ}6$. Finally, the width of the total belt was found to be some 88 km.

10. *The Swedish Expeditions.* With the total belt striking their own peninsula, it is to be expected that the Scandinavian astronomers would be especially interested in this eclipse. In Sweden, for instance, five separate expeditions were sent out from the

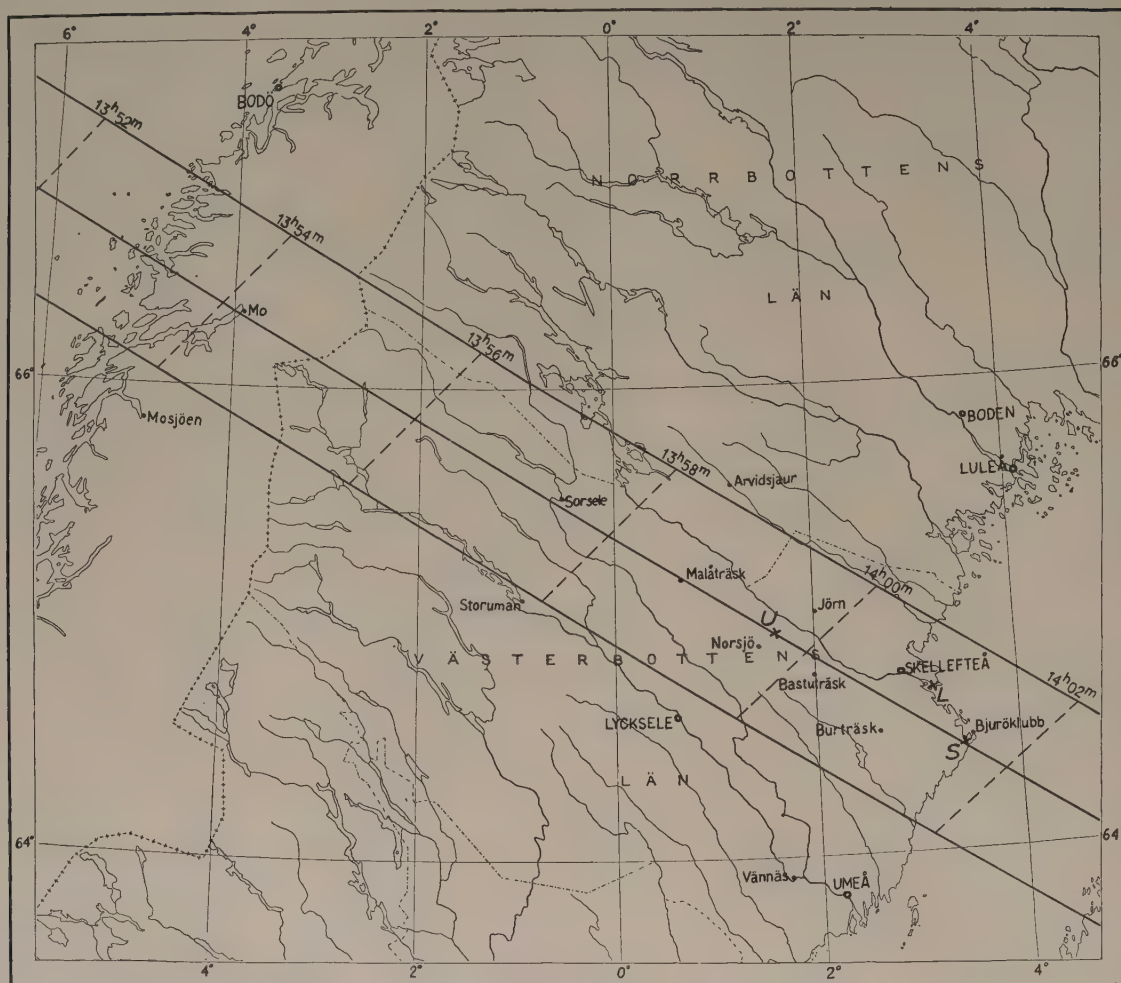


Fig. 1. The path of the total belt over North Scandinavia at the total solar eclipse on July 9, 1945. Crosses marked L (Lund), S (Stockholm) and U (Uppsala) show the positions of the expeditions engaged in contact determinations. The Stockholm Observatory had besides an expedition at Malåträsk, headed by Dr. J. RAMBERG, the Uppsala Observatory, at Bjuröklubb, an expedition headed by Dr. Å. WALLENQUIST. A Norwegian expedition which intended a spectral cinematography for contact purposes was located in Mo (Norway). — Dashed lines are isochrones of maximum eclipse.

astronomical institutes in Lund, Stockholm and Uppsala, of which the two last mentioned were in charge of two expeditions each. A well equipped expedition, headed by Mr. O. WIBERG, was associated with the Stockholm expedition. In addition to the general program of eclipse investigations, different problems of a more special character had been put on the program of research for the individual stations of observation. We shall deal here with one of these, namely the problem of accurate contact determinations, which, in fact, constituted a subject of investigation common to all three observatories. More exactly, one expedition of each institute used the spectrophotometric method,

whereas the Lund expedition also attempted the chronocinematographic procedure, the only one to do so.

In his prediction work for Scandinavia [6], GRÖNSTRAND also included a review of the meteorological conditions to be expected at the critical time of the eclipse. It was found that, within the region concerned, the probability for clear sky increased in the direction west to east. In consequence of this, the region close to the Swedish Bothnian coast was to be preferred for the observations; being rather densely populated, this region was also to be preferred as offering better conditions for the transportation of the eclipse equipment. Generally, the expeditions settled here even though, to avoid the risk of a complete failure, the Stockholm Observatory like the Uppsala Observatory placed one of its expeditions rather a good way up country (Fig. 1).

The observation stations engaged in contact determinations were as follows:

Lund Observatory expedition, in Sävenäs, headed by Professor K. LUNDMARK,

Stockholm Observatory expedition, in Brattås, headed by Professor B. LINDBLAD,

Uppsala Observatory expedition, in Bastutjärn, headed by Professor K. G. MALMQUIST.

11. *The Swedish expeditions (cont.). Weather conditions.* The Lund and the Stockholm expeditions were, during the critical minutes of totality and the entire partial phase as well, favoured with splendid meteorological conditions. The sky, overcast until some hour before the first contact, cleared swiftly owing to a rather strong wind which then gradually died away. The registering flash spectrum equipments and the timing devices worked quite satisfactorily and the program intended could be successfully accomplished. The corresponding Uppsala expedition [22], which had an equipment similar to that of the Stockholm expedition, had its site some 100 km from the Bothnian coast. Unfortunately, it was unable to secure any recordings at all because of a completely clouded sky which, adding to the disappointment, cleared 20 minutes after totality. This misfortune is very much to be regretted since, if successful, the expedition would have contributed towards a considerably wider test basis of the photometric method than could now be established.

12. *Other expeditions engaged in contact determinations.* Finnish geodesists, headed by Professor BONSDORFF, arranged two stations of observation, one at the Bothnian coast (Poroluoto) and one in central Finland (Kangaslampi). On both stations, the recordings were carried out according to the method of direct cinematography and the spectrophotometric method as well. Although the registrations were rather successful at both places, the material secured at Poroluoto turned out to be of a higher quality than that obtained at Kangaslampi.

In Mo in Rana, Norway, an expedition belonging to the Norwegian Astrophysical Institute had intended a spectral cinematography by means of a prism camera. However, it had to give up because of bad weather.

E. HÄLBACH, Director of the Observatory of the Milwaukee Astronomical Society, inspired by LINDBLAD's proposal, undertook privately a timed spectral cinematography from a point located in central Canada. The film obtained showed that the registrations, although in several respects successful, suffer from a rather considerable over-exposure so that accurate results cannot be expected from this material.

Instrumental equipment

13. *General.* The aim of the present investigations was, as related, chiefly to test the spectrophotometric procedure as a means of realizing the geodetic project of BANACHIEWICZ-BONSDORFF. In so doing, an instrumental equipment as uniform as possible for all expeditions was rather much to be desired. The Stockholm and Uppsala expeditions were also in most respects equivalent with regard to their instrumentation. The Lund expedition, unable to procure similar accessories during the comparatively short time of preparation, had for this reason an equipment differing in certain respects from that of the other two.

However, the basic data concerning the registrations were common to all expeditions. So, for instance, effective focal ratio, dispersion and exposure time were practically the same in all cases and in fact directly inferred from those used by LINDBLAD and WIBERG in 1927 (p. 7). The Lund and Stockholm instruments were erected on steady wooden tables with plane oriented parallel to the celestial equator. They were supported at the front end by a short polar axis, whereas, at the lower end, screw adjustments provided for accurate setting on the solar crescent. The Uppsala expedition used for the same purpose an ordinary equatorial mounting.

Concerning the registration of time, the moments of exposure were primarily determined within a provisional time scale, derived by means of a pendulum clock. The provisional times thus obtained were then converted into absolute times in using the rhythmic time signals from the Greenwich Observatory which, by kind courtesy of the Astronomer Royal, Sir HAROLD SPENCER JONES, was transmitted on long wave and short wave from $13^{\text{h}}50^{\text{m}}$ to $14^{\text{h}}12^{\text{m}}$ (U.T.), an interval of time covering the total phase at all stations of observation in Scandinavia and Finland. Moreover, the long wave signal

(GBR, 16 kc sec^{-1}), thanks to the ready courtesy of The Swedish Telegraphic Board, was relayed from several Swedish stations, amongst others Luleå (392 kc sec^{-1}) and Motala (216 kc sec^{-1}). The signal from Motala furnished the Finnish expeditions with universal time, the direct signal being subjected to rather strong atmospheric disturbances.

Lund expedition

14. The expedition, under the leadership of Professor K. LUNDMARK, carried out the observations at Sävenäs which is situated on the coast of the Bothnian Sea, some 15 km southeast of the town of Skellefteå (Fig. 1). The coordinates of the station are given in Table 4, p. 43.

15. *Cinematographic instrument for direct photography.* LUNDMARK, with the intention of rendering possible a comparison between the results derived from direct and flash spectrum procedures at one and the same station, had therefore the expedition equipped with instruments working according to both principles. Although the direct large scale recordings were not particularly successful, we shall nonetheless give a short account of the attempt.

The cinematographic instrument consisted of the Merz refractor of the Lund Observatory equipped with a film camera of a field twice the normal size or, actually $36 \times 24 \text{ mm}^2$. The type of the lens (visual doublet, $f = 429 \text{ cm}$) necessitated the use of a yellow filter (OG 2), which was placed just in front of the image. The camera delivered 8 pictures per second (exposure time $1/70 \text{ second}$) and was loaded with normal diapositive film. The instrument, guided by its ordinary driving mechanism in order to get the highest possible photographic definition, was diaphragmed to a free aperture of 17 cm. The registrations lasted for some 20 seconds at each of the four contacts.

The author, in charge of this instrument, must regret the loss of the third contact, an accident caused by the fact that the proper pointing towards the solar crescent was spoiled when turning the camera around the optical axis to the position angle of the third contact. Consequently, the attempt of timing the eclipse with the accuracy demanded for the geodetic project was unsuccessful. On the other hand, trials made to measure on the recordings secured at the outer contacts, the chord common to the Sun and Moon proved to include so many points of arbitrary interpretation that the results derived could not be trusted very much. The discouraging experiences thus found were mostly due to the fact that for a vanishing chord the shape of the lunar edge will, to a considerable extent, affect the lengths measured. An additional difficulty arises from the



Fig. 2. The Lund Observatory expedition in Sävenäs. The big instrument is the refractor used for a direct cinematography of the Sun near the contacts. The two instruments to the extreme left were operated by Dr. E. HOLMBERG and Dr. A. REIZ who secured blue and yellow plates of the corona respectively. Dr. S. CEDERBLAD, with the third instrument from the left, made a search for paramercurial planets. An additional corona instrument was operated by Mr. G. KLING. The spectral cinematographic instrument is housed just to the right of the refractor.

atmospheric turbulence which, in the case of short exposure times, distorts appreciably the edges of the Sun and the Moon.

16. *Cinematographic instrument for flash spectrum photography.* This was made up of a Zeiss film camera with lens Apotessar ($f = 64$ cm). The attached objective prism of glass SF 2 and refracting angle 26° furnished a mean dispersion corresponding to 3.8 mm between the lines H_γ and H_ϵ . The camera was loaded with panchromatic film of type Agfa Superpan ISS (35 mm, speed 21/10 DIN or about 80 ASA in daylight) and had a nearly constant output of 24.7 exposures per second. The time of exposure was $^s.013$, the focal ratio 1:12. During the short time of registration, some 25 seconds at each contact, the instrument remained stationary but had, of course, to be readjusted in between the contacts. The recording started 10^s before the (predicted) second contact and ended 10^s after the third.

Since, according to the prediction data, the second and third contacts should occur at position angles differing by 138° (in consequence of the location off the central line, Fig. 1), the prism's refracting edge had to be turned through this angle in between the two contacts. Mr. N. HANSSON, assistant at the Lund Observatory, solved this problem ingeniously by making the cinematographic instrument rotatable as a rigid unit around an axis which should be pointed towards the Sun. Accordingly, what had to be done

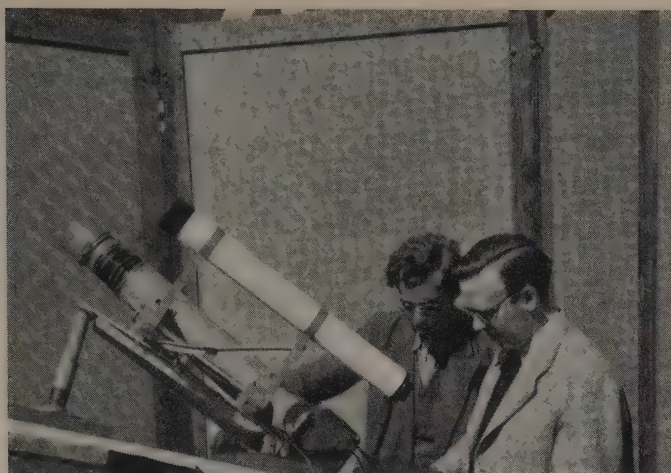


Fig. 3. Mr. N. HANSSON (left) and Mr. L. STIGMARK at the prism camera of the Lund expedition.

during the remaining seconds of the total phase was merely to revolve the instrument 138° around this axis.

The cinematographic instrument was constructed and operated by Mr. HANSSON.

17. *Timing device.* The procedure followed in timing the direct and spectral recordings was worked out by Mr. L. STIGMARK, assistant at the Lund Physical Institute, who also joined the expedition in his capacity of an expert on electronics. Mr. HANSSON likewise contributed essentially to the practical accomplishment of the device, the accessories of which (except for the oscillograph) were built in the Lund Physical Institute under the supervision of Mr. STIGMARK.

The arrangement was based on using an oscillograph with two rays. Of these, the *first ray* registered the swings of a pendulum clock, the moments of exposure of the individual cinematographic records of both cameras and the direct long wave rhythmic signal GBR, 16 kc sec^{-1} , whereas the *second ray* was performing oscillations caused by an impressed a. c. of $50 \text{ cycles sec}^{-1}$. The movements of the rays were followed in imaging the luminous points on a film, running continuously with a speed of about 12.5 cm.sec^{-1} ; any possible fluctuation in this speed was controlled with the auxiliary frequency impressed on the second ray.

The pendulum impulses were generated by means of a light beam, sweeping over the aperture of a photoelectric cell at every position of rest of the pendulum. This was effected by imaging, with the aid of a simple lens, the spiral filament of a glow-lamp upon the photocell and by letting the convergent beam be reflected from a small plane

mirror attached to the pendulum itself. The lamp was placed some 3 dm beside the pendulum bob, whereas the photocell rested some 2 dm below. The impulses produced were, before recording, strongly amplified. — The pendulum clock, Kessels No. 1418, was housed in a cover of a wood-fiber product in order to render it insensible, as far as possible, to fluctuations in the temperature. This measure proved to be a happy one since thereby, in spite of rather strong variations in temperature, the performance of the clock turned out to be quite excellent. A concrete pillar provided a steady suspension of the astronomical clock.

The impulses indicating the different exposures were effected by an eccentric wheel on the axis of the rotating camera sector which, at the very moment of exposure, touched a metal spring connected to the oscillograph, thus resulting in a synchronous deflection of the ray of electrons. To recognize the time mark corresponding to any given exposure, marks of orientation between the time film and the film containing the exposures had to be introduced. This was achieved in different ways for the direct camera and the prism camera. In the first case (direct cinematography), part of the recorded solar image was hidden by putting into the light beam, just in front of the film, a thin metal disc. This was made repeatedly and for very short moments by pressing the key of an electric relay which operated the disc and which, moreover, acted upon one ray of the oscillograph. The result was a deflection of the ray, occurring at the same time as the recording of the shadowed solar image. In the second case (flash spectrum cinematography), the same effect should be attained by replacing the occulting sheet by an electric lamp, the light glimpses of which — caused by operating the key of a second relay — should be photographed on the spectral records. Unfortunately, the arrangement did not function according to plan, probably because of a short-circuit which spoiled the lamp. The exact one to one correspondence could, however, be established without ambiguity at the second contact by comparing the time marks and the exposures throughout the registration. On the other hand, at the third contact, the situation was not so favourable since the spectral film had come to an end before the moment at which the camera stopped. The remaining possibility of using the very start of the recording at this contact as a mark of orientation proved to be unable to settle the correspondence uniquely. The accepted moments of exposure might accordingly be liable to a certain error, the maximum value of which has been estimated as .520 seconds (cf. p. 78).

The direct rhythmic signal, — that is, not the relayed one, — was recorded immediately before the second contact and immediately after the third, in both cases in the



Fig. 4. Copies of the oscillographic registration of time at the Lund expedition. Sharp peaks represent the second strikes from the pendulum clock; the sinusoidal curve represents the controlling frequency. *Above:* registration of the exposure moments of the two movie cameras (short impulses refer to the flash camera) *Below:* registration of rhythmic signals from Greenwich Observatory.

course of some 2 minutes. The dashes of the signal, corresponding to integral minutes, served the purpose of yielding the zero point of the time registration.

The Lund timing device was operated by L. STIGMARK.

18. *Accuracy in the determination of time.* The oscillographic registration of time turned out to give an excellent accuracy. Within the provisional time scale, furnished by the pendulum clock and the controlling frequency, the moments of exposure could easily be settled with an exactitude of .001 second. The conversion of this time into universal time could also be carried out in a satisfactory manner as shown by the figures in Table 1.

Table 1

Clock corrections to the rhythmic signal

Beat number (from 13 ^h 59 ^m U.T.)	Corr.	Beat number (from 14 ^h 02 ^m U.T.)	Corr.
4	+0 ^s .002	1 (long)	0 ^s .000
5	+ .004	2	+ .004
7	+ .003	4	+ .004
9	.000	6	+ .004
16	+ .002	10	+ .005
19	+ .001	18	.000
20	+ .002	36	+ .002
Mean	+0 ^s .0020	Mean	+0 ^s .0027
	± .0005		± .0008

The clock corrections given in this table have been derived from a number of rhythmic beats which apparently were unaffected by atmospheric radio noise. In doing so, the provisional time has been assumed to be defined by the upper, very sharp peaks of the pendulum marks, a choice which by mere chance rendered the corrections to be of

an almost negligible value. It is found that the corrections, within the limit of their accuracy, come out with identical values from the two comparisons. This is in agreement with the fact that the clock rate was quite negligible at the time in question.

The time furnished by the GBR-signal has to be corrected for the time of propagation and for errors affecting the moments of transmission (cf. p. 26).

Stockholm expedition

19. The expedition, under the leadership of Professor B. LINDBLAD, had its site at Brattås which is very near to the Bothnian coast and about 33 km in a southeastern direction from the town of Skellefteå (Fig. 1, p. 12). The coordinates of the station are given in Table 4, p. 43.

In addition to the cinematographic recording secured at this ground station, a flash spectrum cinematography was undertaken from an air-craft of the B3 type which was kindly put at the expedition's disposal by General B. NORDENSKIÖLD, commander-in-chief of the Royal Swedish Air Forces.

Several members of the Office of the Swedish Geographic Survey were likewise engaged in the carrying through of the elaborate program for a precise determination of contacts. Dr. E. BERGSTRAND, with the aid of a visual refractor ($f = 4.23$ m), secured a number of plates with an aim at determining in this way the relative position between the Sun and Moon. Dr. B. AURELL, by means of a transit instrument, observed a number of clock stars for the purpose of determining the astronomical coordinates of the observation site. Finally, Mr. E. V. U. LANDBERG and Mr. A. LEIJONHUFVUD carried out special investigations from places situated close to the borders of the total belt. — A report of the activities of the Stockholm expedition is presented in [20].

20. *Cinematographic instrument for flash spectrum photography.* The instrument was a reflecting telescope with a parabolic mirror ($f = 101$ cm, free aperture 20 cm) in conjunction with a movie camera of the Eyemo type attached to the Newton focus. The objective prism of glass SF 2 and refracting angle 14° delivered a linear dispersion corresponding to 3.6 mm between the lines H_γ and H_ϵ . The parabolic mirror and the objective prism as well were manufactured for this particular purpose by Mr. E. AULIN. The movie camera was kindly lent by the moving picture company Svensk Filmindustri, Stockholm. The camera was loaded with film Kodak Plus-X, Panchromatic (35 mm, speed 50 ASA in daylight). The spectral registration started 10^s before the second contact, continued during the total phase and ended 10^s after the third contact; the camera was however put into action 20^s before the beginning of the cinematographic registra-



Fig. 5. The Stockholm Observatory expedition at Brattås (from the air). LINDBLAD's cinematographic instrument, the spectrograph for registration with moving plate and Dr. ÖHMAN's instruments for polarimetric measurements were housed in the large pavilion. Mr. WIBERG's instruments were housed in the small pavilion to the left. Dr. AURELL's and Dr. BERGSTRAND's instruments were located some few hundred yards to the right of the central pavilion.

tions which were allowed to take place by uncovering the aperture of the parabolic mirror. The instrument was shifted to the third contact at the middle of the total phase. The time of exposure was $0^s.01$, the effective focal ratio $1:14$ (with due regard to the shadowing effect of the secondary).

Since the observation site was very close to the central line and, accordingly, the difference in position angle between the inner contacts almost exactly 180° , LINDBLAD preferred to have the prism unturned between the contacts. The advantage of such a procedure is obvious from several points of view; we shall have reason later on to make use of one important consequence: namely that, in doing so, the directions of dispersion will differ by 180° at the two contacts.

The instrument was constructed under the supervision of Professor LINDBLAD who also operated it during the eclipse.

21. *Flash spectrum registrations by means of moving plate.* As has been stated before, LINDBLAD suggested a spectral photometry not only with a view of timing the eclipse with great accuracy but also because of the fact that a high speed recording of the swiftly proceeding phenomena of the flash spectrum would be of great value from an astrophysical point of view. The cinematographic procedure renders possible a close-up study of the chromosphere and the extreme layers of the photosphere along a fairly broad portion of the solar periphery. The moving plate technique, introduced by

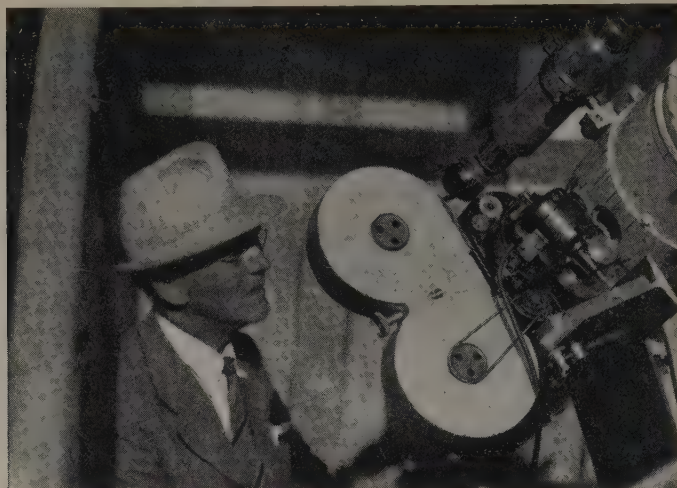


Fig. 6. Professor LINDBLAD at his cinematographic instrument.

W. W. CAMPBELL, makes possible a similar study at a selected point of the solar edge with a larger dispersion.

To make the astrophysical researches more complete, LINDBLAD therefore undertook, in cooperation with Dr. R. BRAHDE, Oslo, a timed moving plate recording using a three prism spectrograph — belonging to the Grubb 40" reflector of the Stockholm Observatory — attached to a Cassegrain of 5 m equivalent focal length and 25 cm free aperture. The instrument which was mounted horizontally and fed by a coelostat produced a dispersion amounting to $30 \text{ \AA}$ per mm in the blue part of the spectrum. The panchromatic plate (Astra III) was allowed to move with a speed of 6 mm per second. Time marks on the plates (one for each contact) were achieved in letting the rod of a metronome swing in front of the slit of the spectrograph; this metronome when passing its zero position made contact with a spring connected to the oscillograph. A solar reference spectrum was exposed on the moving plates in order to render possible a determination of the spectral energy distribution at the Sun's limb.

22. *Timing devices (general).* The Stockholm expedition was provided with extensive arrangements for the purpose of timing accurately the cinematographic records. This aim was, in fact, attained in two different and independent ways. More strictly speaking, the moments of exposure, within the provisional pendulum time obtained from a Richter clock, were established both by means of oscillographic registrations and by means of registering photographically the pendulum swings on the spectral exposures themselves. Conversion of this time into absolute time was likewise achieved in two different ways,

namely in recording the rhythmic signals by oscillograph and by electromagnetic oscillator. Moreover, to avoid the risk of losing the proper connection to universal time in the case of failing records of the time signal, observations of star transits should provide the expedition with time of highest possible accuracy. Dr. B. AURELL, in charge of the transit instrument, was also able to secure for both positions of the instrument a clock star during the total phase. Fortunately, however, the registrations of the time signals were very successful so that both expeditions could take advantage of a common reliable time basis.

The oscillographic device and the receiving radio sets for long wave and short wave were operated by Dr. T. ELVIUS, assistant at the Stockholm Observatory, who moreover was in charge of most of the preparatory work of procuring accessories of different kinds, arranging with antennas etc. Some technical problems were solved in consultation with experts in the field of electronics and radio communications. In particular, valuable assistance was afforded by Mr. W. STOFFREGEN of the Institute of High Tension Research at Uppsala, Mr. S. GEJER of the Swedish Telegraphic Board and by Mr. E. AULIN and Mr. R. SCHÖLDSTRÖM of The AGA Baltic Company Ltd in Stockholm. The two last mentioned also joined the expedition for the purpose of carrying out photographic studies of the corona.

23. *Timing devices (details).* A simple scheme of the principles used by the Stockholm expedition is shown in Fig. 7. There, the procedures briefly outlined in the foregoing section have been grouped so as to constitute two complete timing devices, working

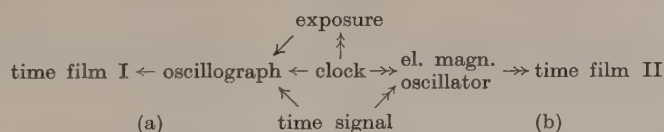


Fig. 7. Principle of the timing devices used by the Stockholm expedition.

quite independently of each other. This way of considering the problem, though not necessary from a methodical point of view, facilitates to a certain extent the description of the arrangement as a whole. The devices in question, distinguished by the two kinds of arrows, will be described below as (a) and (b).

(a) *Procedure I* (ordinary arrows), based entirely on registrations by means of a two ray oscillograph, was similar to that employed by the Uppsala expedition at the same eclipse. It had been designed by K. G. MALMQUIST, H. NORINDER and W. STOFFREGEN who have described it elsewhere in full detail [21]. The marks of orientation were however arranged

in a different manner (see below). It differed from the similar arrangement built by STIGMARK for the Lund expedition particularly as regards the manner in which the pendulum impulses were generated, those being produced here by a voltage induced by an electric coil at every zero position of the pendulum.

The different impulses were distributed on the rays as follows: on *the first ray* the pendulum impulses, the time signals and the metronome impulses timing the moving plate recording; on *the second ray* the camera impulses and a controlling frequency of 50 cycles sec^{-1} . All these signals were recorded on a film (time film I) running with a speed of 5.5 cm sec^{-1} . This speed was subjected to a steady control by the auxiliary frequency impressed on the second ray.

When, as was actually the case, the camera signals and the time signals act upon different rays, the position of synchronous points has to be determined; the relative position may, in fact, be adjusted arbitrarily along the time axis of the oscillograph. This determination was carried out immediately before the eclipse by letting one and the same impulse act upon both rays and by measuring, on the film record, the projected relative distance along the time axis. The result, converted into time, proved to be .⁵⁰⁰⁸.

Marks of orientation were established by means of an auxiliary pendulum, suspended from the door of the pendulum clock and shadowing — through a metal sheet attached to the rod — the steady light signal of Procedure II (compare (b)) at every zero position. This pendulum was allowed to swing every fifteenth second, thereby striking on and rebounding again from an elastic spring. The contact was recorded on the oscillograph. Thus, at these moments, the steady image on the spectral film disappeared at the same time as the ray of the oscillograph underwent a deflection from its normal position. The procedure was repeated several times to avoid the risk of having the contact happening between the moments of two consecutive exposures.

(b) *Procedure II* (double-headed arrows). This was constituted by transferring optically the pendulum swings to the spectral exposures; the correction of the astronomical clock was determined in recording on a separate film (time film II) the pendulum swings and the rhythmic signals (LINDBLAD).

An illuminated slit, placed in front of the oscillation centre of the pendulum, was imaged beside the flash spectra. In fact, two images were produced, one steady and one performing oscillations (about the fixed image) in phase with those of the pendulum itself (Figs. 13 and 14). This was achieved by letting two pencils of light, both emerging from the same slit, be reflected from small prisms, of which one was fixed, whereas the other was swinging with the pendulum. Both rays were reflected through these prisms into the

axis of the instrument and finally forwarded to the film by means of an additional prism. Evidently, measurements of the relative distance between the two slit images will furnish the moments of the actual exposures, provided the zero-point of the time scale has been established in one way or another.

This was arrived at as follows. A narrow pencil of light, after reflection from a small mirror, was allowed to pass through an objective attached to the pendulum. This objective focussed the light pencil on a second running film (time film II). The result was a sine curve arising from oscillations of the pendulum. The small mirror, which was fastened to the magnetic core of an electric coil, could perform rapid oscillations by impressing on the coil an alternating current of proper frequency. This electromagnetic oscillator, when connected to a receiver tuned to a station performing time signals, was accordingly able to record the beats of the signal as superimposed on the sine curve just mentioned. The absolute time of any beat is readily derived either by starting from the beat of any known integral minute or by interrupting briefly, at known seconds of the pendulum clock whose correction is known approximately, the light signal photographed on the time film II. Finally, to link together the absolute time scale thus derived and the relative time scale impressed on the spectral records, one merely needs a mark of synchronization between the two film recordings. This was made by interrupting simultaneously and for very short periods the electric currents which fed the two lamps producing the recorded light signals.

24. *Reception of the time signals. Accuracy in the determination of time.* The signal from the Greenwich Observatory was recorded, in several stages, from the stations GBR (16 kc sec^{-1}), GIH ($10650 \text{ kc sec}^{-1}$) and Luleå (392 kc sec^{-1} , relayed). The strength of the direct GBR signal was, however, sometimes very faint and during the total phase this signal had to be replaced by the relayed signal. ELVIUS remarks, on the other hand, that the level of the atmospheric noise was rather low as compared to that observed on previous occasions.

LINDBLAD, from a study of the accuracies attained with the two procedures, finds both of them to be quite satisfactory, that is to say, the individual exposures could be timed with an exactitude greater than $0^s.01$. He also judges the oscillographic method to be slightly more accurate than the other.

The clock corrections, at moments near the two contacts, were found by ELVIUS to be:

Correction	Signal	Cont.
$+ 0^s.2686 \pm 0^s.0013$	GBR	second
$+ .2516 \pm .0011$	Luleå	third

The mean rate of the pendulum clock, as derived from daily comparisons, amounted to $0^s.0011 \text{ min}^{-1}$ (retarding). Correcting for this rate, the remaining difference between the two signals comes out to be $0^s.0181$, a figure which obviously must be ascribed to the delay of the relayed signal relative to the direct one. Now, according to information from The Swedish Telegraphic Board, the time of propagation from Rugby to Brattås should be for the direct signal $0^s.0062$ whereas, for the relayed signal, the corresponding time should be $0^s.0250$, arrived at as follows:

Rugby—Enköping (wireless)	$0^s.0046$
Enköping—Luleå (by wire)	$.0200$ (measured)
Luleå—Brattås (wireless)	$.0004$

The relative delay, accordingly, will be $0^s.0250 - 0^s.0062 = 0^s.0188$, a figure (within the limits of the mean error) agreeing rather well with that derived by ELVIUS, or $0^s.0181$. The agreement is somewhat better than could be expected from the fact that the appearance of the two signals was not quite the same; the strength of the retransmitted signal was, of course, much greater than that of the other which may have contributed toward the slightly differing character of this signal.

Certain indications from the oscillographic registration of time pointed to a slightly fluctuating clock rate, the amplitude amounting to some thousandths of a second. Such fluctuations, if real at all, are of course beyond all importance for the present problem.

The registered times were corrected with $+ 0^s.058$ for a delay in the time of transmission, as communicated by the ASTRONOMER ROYAL.

CHAPTER III

The lunar contour as obtained from ordinary large scale photographs

25. *General.* Remembering that the topography of the lunar limb inevitably forms a severe obstacle in the problem of accurate contact determinations, one evidently has to look for the possibilities which might render available a precise knowledge of the irregularities in question. Now, as will be proved in the course of the present investigation, the photometric procedure allows at the same time the determination of the acting lunar contour with a considerable accuracy. Consequently, in this respect, the photometric method is in no need of any additional data of this kind. On the other hand,

some interest might be associated with the carrying out of a comparison of the profile resulting from this and other independent procedures.

For an investigation like that, there remains merely the possibility of deriving the shape of the lunar profile from a study of large scale photographs of the Moon. Such photographs may be obtained either at Full Moon or at solar eclipse when the lunar edge is seen projected against the background of the solar disk. The lunar contour may be obtained from direct measurements of the record by means of a measuring machine; another procedure, which seems to be very promising indeed, has recently been worked out by C. B. WATTS and A. N. ADAMS [33] at the U. S. Naval Observatory, Washington. The latter method consists in determining the shape of the contour by following a line of constant density at the marginal zone of the photographic records (cf. p. 82).

26. *The lunar contour according to investigations by F. HAYN.* The most complete discussion ever undertaken about the topography at the limb regions has been given by F. HAYN [10]. The results of his researches have been condensed in the form of a topographic map which gives the isohypses within a marginal belt broad enough to cover all occurring states of libration. The step in height between two consecutive isohypses amounts to 0".2.

For the construction of this map, HAYN started by securing a great many photographic records of the Full Moon when this was in different states of libration. The scale was fairly large, corresponding to a focal length of 3.6 m. All records were then investigated by means of a comparator with special regard to the limb irregularities and the obtained heights plotted on a map at their proper places on the lunar surface. To find these places, HAYN made use of a system of spherical coordinates, the positive pole of which is the point of intersection between the first meridian and the equator of the Moon. P , the longitude, is reckoned from the lunar north pole in the same direction as the position angle; D , the latitude, is reckoned positive in the direction of the observer. The region investigated by HAYN is accordingly symmetrical around the great circle $D = 0$. — The work has been carried out in a very careful manner. Thus, for instance special measures were taken to account for effects which might arise from the presence of a varying albedo at the lunar limb.

To find the regions of the marginal belt which contribute toward the actual shape of the Moon's edge at a certain position angle p , one has to find the point (P, D) on the lunar surface where the observer's line of sight touches the lunar sphere. The locus of this point, for running p , forms on the topographic map a curve which evidently

signifies the points of contact between the Moon and the tangent cone with apex situated at the observer. HAYN [10, IV p. 15] has devised a simple and very rapid procedure, by means of which (P, D) may be derived with an accuracy of some 0.1° in each coordinate. Since this accuracy is quite sufficient for the present purpose, the procedure in question has been followed here.

The total libration (the negligible physical libration excluded) in selenographic longitude (l') and latitude (b'), as well as the position angle C of the first meridian, were found from *The Berliner Astron. Jahrbuch 1945* to be:

$$l' = + 4.65$$

$$b' = - 0.22$$

$$C = + 8.73$$

These figures, which properly speaking represent the values of the libration constants for the Lund expedition at 14^h U.T. (the maximum phase occurring 1 minute later), are at the same time representative also for the Stockholm expedition; as a matter of fact, the differences in the coordinates of the observation sites (Table 4, p. 43) are actually so small that the libration constants, as computed for the same moment, come out in both cases with the values given above.

On the basis of these constants, the coordinates P and D have been computed for every fifth degree of position angle; after this, by means of the corresponding curve laid out on the topographic map, the heights have been determined at every degree of the lunar periphery. In doing this, the fact was taken into account that the observed profile will result not only from the topography at this marginal line but also, eventually, from the topography of regions close to it. This was achieved by adopting as a final value of height, at any given position angle, the maximum value of the expression

$$h_v - R''(1 - \cos v)$$

in which h_v means the height at a point situated at the lunocentric angular distance v from the marginal curve and R'' the angular radius of the Moon (expressed in seconds of arc). The last term gives the perspective depression of a certain object when referred, along the line of sight, to the very limb. However, since this depression already for $v = 2^\circ$ takes a value about equal to the average amplitude of the limb irregularities corrections need to be applied only occasionally for greater values of v .

The lunar contour thus obtained is reproduced in Fig. 11. Later on, we shall find reason to make a provisional use of HAYN's contour when determining the position of the lunar centre by means of the photometric profiles.



Fig. 8. One of the plates secured by BERGSTRAND (approximate natural size).

27. *The lunar contour as obtained from photographic records of the actual eclipse.* As stated before, large scale records of the partial phase were secured by Dr. E. BERGSTRAND, using a visual refractor of $f = 4.23$ m. As a rule, four crescents were recorded on every plate (Ilford Halftone 16×16 cm, orthochromatic), the time intervals between successive exposures being $1^m 20^s$. With the objective diaphragmed to $1:50$ and with a strongly absorbing yellow filter (glass plate covered with a layer of vaporized iron) just in front of the plate, the proper exposure time came out to be $0^s.25$. The instrument remained in a fixed position but the daily motion was accounted for by moving the plate with the proper speed. The moments of exposure were obtained by means of a chronograph. — The plates, of which one is reproduced in Fig. 8, have been measured by BERGSTRAND, mainly for the purpose of establishing accurately the relative position between the Sun and Moon. The images were of a very fine definition and, accordingly, well adapted for investigations of the lunar limb. The present author was also allowed to use the plates for this purpose.

The plates selected for use were investigated by means of the Gaertner measuring machine of the Stockholm Observatory. The procedure employed was measuring the distances from a well defined preliminary centre to the lunar edge for every degree along the lunar periphery. This centre was placed very close to the true one and consisted of a minute hole in the emulsion made with the aid of a very sharp needle. It is well known that at measurements of this kind the observer has to be very careful for any possible change in his perception of the limb, which is always more or less blurred. In order to reduce such errors as far as possible, the measurements should be repeated in the opposite direction of position angle. The repeated measurements were,

however, restricted to every other degree of the lunar periphery. The orientation of the contours was determined from the direction of the cusp chords and from the direction of the lunar daily motion as well.

Some data bearing upon the measured plates have been listed in Table 2. Of these plates, those with numbers 3 and 4 were taken before the total phase whereas the remaining ones were secured after totality. The Roman figures refer to the number of the crescent selected for investigation.

Table 2
Data concerning the measured plates

Plate No.	Moment of exposure (U.T.)	Measured	Remeasured
3 II	13 ^h 51 ^m 52 ^s .8	Feb. 1948	
4 III	13 58 23.4	"	
5 IV	14 07 03.0	"	Aug. 1948
6 IV	14 12 33.0	Aug. 1948	
8 II	14 22 04.2	Feb. 1948	Aug. 1948

To the measured values of the radii vectors, corrections must now be applied, when necessary, for distortion of the recorded image through projection on the plate (normal distortion), for differential refraction, for refraction in the glass filter and, finally, for the slightly incorrect position of the preliminary centre with respect to the true centre.

(a) Normal distortion means that the angular and linear scale of the projected image are not quite proportional to each other. In particular, such effects may arise from a tilt of the plate towards the optical axis and from the fact that the lunar centre does not coincide with the plate's optical centre (intersection of the optical axis with the plate). Actually, as shown by BERGSTRAND, the tilt mentioned was less than 2'. Further, so far as the crescents measured here are concerned, the distance between lunar centre and optical centre on the plate was at its most 19.5 mm. It is easy to show that under these circumstances and when using the actual focal length of 4.23 m, normal distortion does not affect the measured radii by as much as 0.001 mm. Accordingly, since this figure equals that of the accuracy of the measurements, the errors produced by the image's projection on the plate can be neglected.

(b) The corrections for the distortion of the lunar edge, which is due to refraction in the Earth's atmosphere, have been obtained from L. DE BALL's *Refraktionstafeln* (Leipzig 1936). The visual refraction tables have been used (yellow filter) although this measure,

in the present context, is not really necessary. Table 3 gives the corrections in millimeters (scale 0.0205 mm per second of arc) which should be added to the measured radii of the various zenith distances z_{rad} . Listed in the table are, however, only the quantities bearing upon the first and the last of the four plates, while for the remaining ones the corresponding values have been obtained by interpolation.

(c) The influences arising from the glass filter are easily derived from an inspection of Figs 9 a and 9 b. In the first of these, O is the objective of the refractor and PP' the displacement of P due to refraction in the glass plate; the remaining notations are self explanatory. Obviously, denoting by n the refractive index,

$$\begin{aligned} PP' &= d(\operatorname{tg} i - \operatorname{tg} b) \\ &= d \cdot \frac{PQ}{F} \left(1 - \frac{1}{n} \right) \\ &= c \cdot PQ \quad (c = \text{const.}) \end{aligned} \quad (1)$$

which is true since the angle i is very small. Now, looking at Fig. 9 b which is merely the first figure viewed from above, one gets the desired correction Δr as equal to $PP' \cos \mu$. Accordingly, from (1)

$$\Delta r = c \cdot PQ \cdot \cos \mu,$$

or, from Fig. 9 b

$$\Delta r = c(r + QC \cos \varphi)$$

where QC is the distance between the plate's optical centre and the centre of the lunar image. With $r = 19.8$ mm, $d = 2.21$ mm, $n = 3/2$ and $F = 4230$ mm, one gets (in mm)

$$\Delta r = 0.0034 \left(1 + \frac{QC}{r} \cos \varphi \right) \quad (2)$$

which should be added to the measured values of r .

d) After applying the corrections treated above, the lunar mean contour should now be of a circular form. To find this mean level and, then, from that the true shape of the lunar contour, consider Fig. 10, where the following notation has been employed:

Table 3

Refraction corrections for plates No. 3 and 8.
 z_{rad} = angle between radius vector and direction to zenith, $z_{\odot}$ = Sun's zenith distance.

z_{rad}	Plate No. 3	Plate No. 8
	$z_{\odot} = 52^{\circ} 31'$	$z_{\odot} = 55^{\circ} 22'$
$0^{\circ} \quad 180^{\circ}$	+0.0148 mm	+0.0169 mm
10 170	145	165
20 160	136	155
30 150	125	141
40 140	109	122
50 130	93	103
60 120	78	83
70 110	66	68
80 100	58	58
90 90	55	55

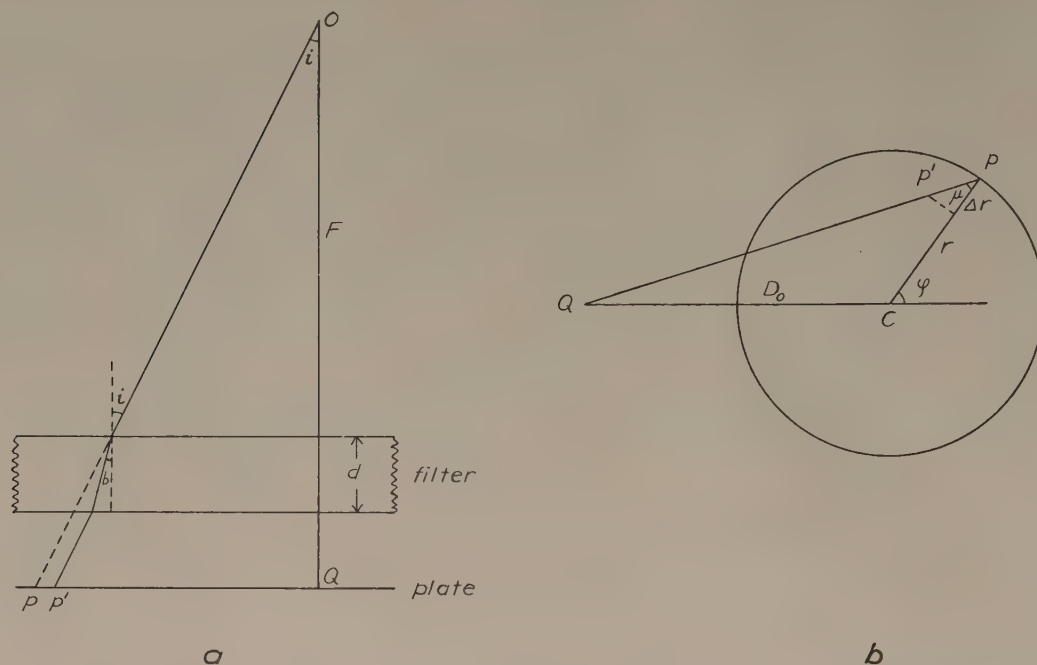


Fig. 9. Figures showing the effect of the glass filter on the distances measured on the plates.

C, C_p = true and preliminary centre resp.,

N = fundamental (north) direction in the polar coordinate systems,

d, ϕ = polar coordinates of C with respect to C_p ,

r, p = polar coordinates of L (on the limb) with respect to C_p ,

R, r = lunar mean radius and measured radius, resp.,

h = height of L above mean level,

$\mathfrak{P}$ = the supplement of $(\phi - p)$,

ε = the angle CLC_p .

The following simple relations are easily obtained from the triangle CLC_p :

$$(R + h) \cos \varepsilon = r + d \cos (180^\circ - \phi + p); \quad (3)$$

$$(R + h) \sin \varepsilon = d \sin \mathfrak{P}. \quad (4)$$

Developing in (3) $\cos \varepsilon$ to the second power of ε , and putting

$$\cos p = a, \quad d \cos \phi = x,$$

$$\sin p = b, \quad d \sin \phi = y,$$

we get from this relation

$$ax + by + R = r + \frac{R}{2} \varepsilon^2 - h \quad (5)$$

in which the small term $\frac{1}{2} h \varepsilon^2$ has been omitted. Finally, replacing from (4) the small

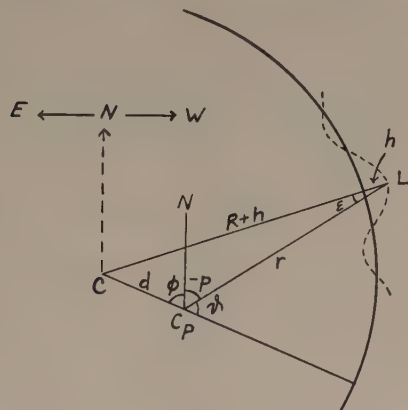


Fig. 10. Determination of centre and mean level of the measured contours.

angle ε by $\frac{d}{R} \sin \vartheta$ and putting

$$R = z + R_0,$$

$$r = r' + R_0,$$

where R_0 is an approximate value of R , one obtains from (5)

$$ax + by + z = r' + \frac{d^2}{2R_0} \sin^2 \vartheta - h. \quad (6)$$

This relation (6) has been used as an equation of condition for the purpose of finding, by means of the method of least squares, the limb irregularities h sought for. A preliminary solution, based on some ten equations only, furnished the value of the small second term which enters in the right member. The final solution was then carried out on the basis of all the radii which had actually been measured (some 150 for each crescent).

The contours arrived at in this way are given in Fig. 12; the two profiles covering the range $18^\circ < p < 170^\circ$ refer to the eastern limb (plates No. 3 and 4), whereas the remaining two bear upon the western limb (plates No. 5 and 8).

28. *Intercomparison of the derived contours.* The curves in Figs. 11 and 12 allow us immediately to study the agreement between the lunar profiles as obtained from HAYN'S topographic map and from the measured records of the eclipse. Also, the two independently stated profiles in Fig. 12 allow a rough valuation of the errors which affect the results from the last mentioned procedure.

(a) A comparative examination of the measured contours reveals that the difference in height, Δh , at corresponding points is far from being distributed at random when con-

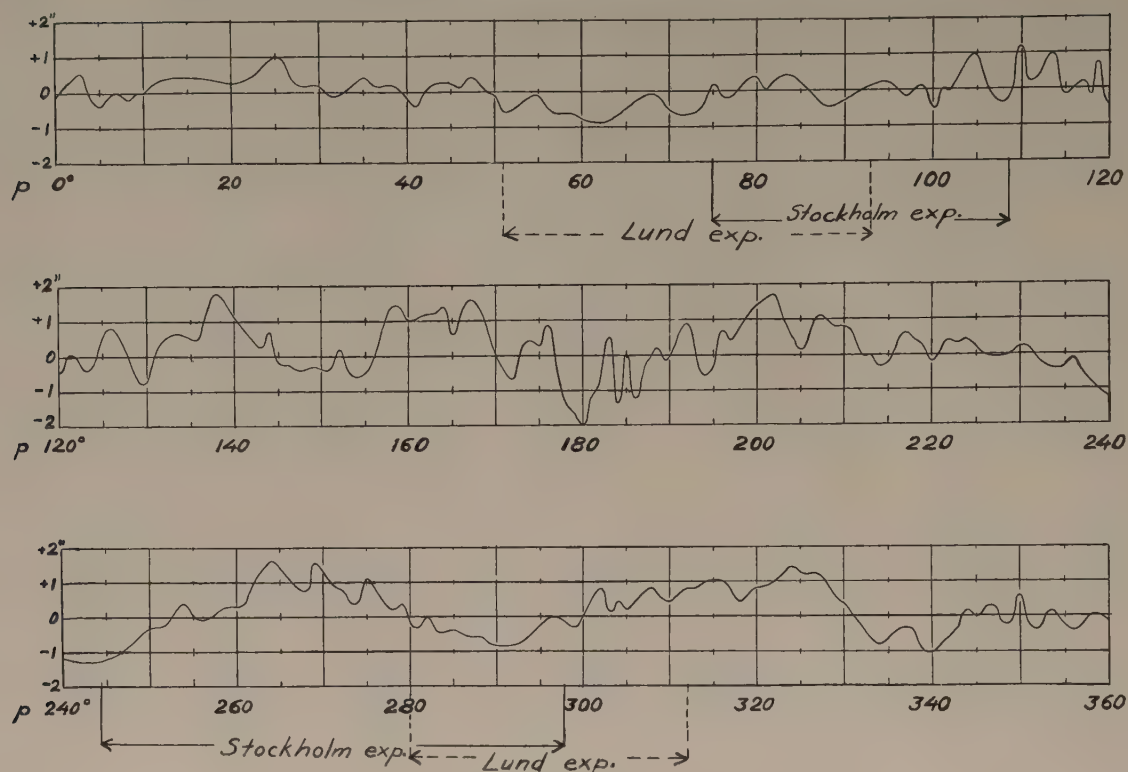


Fig. 11. The lunar profile according to HAYN's topographic map. Horizontal scale gives position angle along the lunar periphery, vertical scale height in second of arc above mean level. The portions marked "Lund exp." and "Stockholm exp." refer to the contours dealt with in the present photometric investigation.
Second contact above, third below.

sidered with running values of the position angle. On the contrary, long sequences occur where the sign of Δh is unchanged, e.g. within the regions 234° — 264° and 264° — 294° . The discrepancy in this interval — the systematic difference in level amounts to as much as $1''$ or even more — was so striking that it prompted a renewed measuring of the plates concerned, as has already been indicated in Table 2. In doing so, new preliminary centra were introduced; moreover, one crescent (plate No. 6) was subjected to measurements besides. The results from this investigation were as follows: (1) the re-measured profiles appeared with the same shape as before, (2) the third contour, likewise, showed considerable systematic divergencies from the two other. Accordingly, the departures in question are not likely to result from errors committed at the measurements of the plates. The mean radii R derived from the three contours are given below; actually, the figures refer to the values at 14^h U.T., that is to say, corrections have been applied for the decrease in apparent radius which, as a result of the slightly changing topocentric lunar parallax, amounted to $-0''.028 \text{ min}^{-1}$.

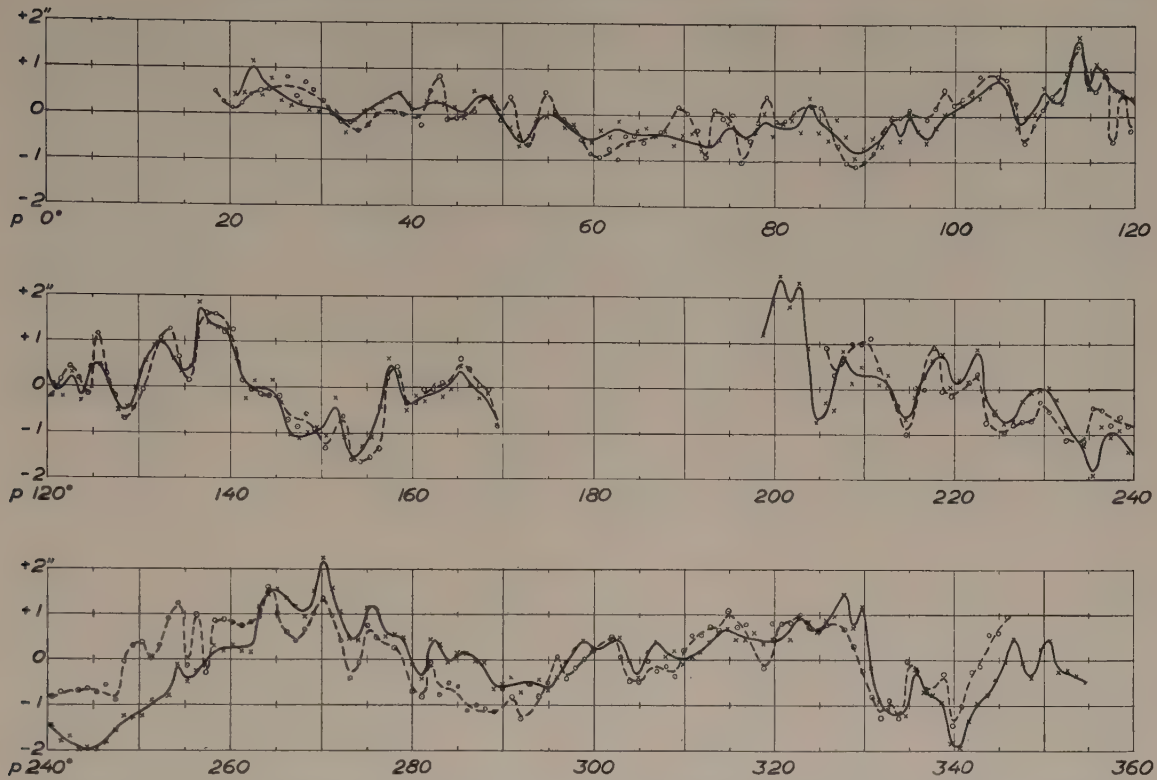


Fig. 12. The lunar profile according to direct measurements of plates taken during the partial phase (cf. Fig. 11).

Plate No.	R (mm)
5	19.670 ± 0.005
6	$19.665 \pm .006$
8	$19.660 \pm .005$

To explain the different shape of the western profiles, there now remain two possibilities: refraction, or a not quite proper speed of the recording plate. The latter alternative is ruled out by the fact that the measured crescents all showed a satisfactory definition and, furthermore, by the fact that the obtained R -values display a reasonable constancy.

The contours representing the eastern limb display typical effects from accidental refraction such as may be observed at the "boiling" solar limb. It is well known, however, particularly from investigations by F. SCHLESINGER [29], that the star images within a small region of the sky are able to undergo not only rapid individual fluctuations but also can deviate simultaneously and along parallel paths from their mean positions by as much as $1''$. Group displacements of this kind may last for even a minute of time. This is obviously the reason why the western profiles behave so differently. It is also

to be expected that the atmospheric inversion layers, where these disturbances are assumed to have their origin [5], will be subjected to rather abnormal conditions at a solar eclipse since, at that time, the insolation on the layers must undergo considerable alterations.

To find a rough measure of the amount with which accidental refraction affects the stated heights, denote by h_1 and h_2 the heights at corresponding points on the limb and by $\varepsilon(h)$ the mean error in one determination of height. From the two contours representing the eastern limb, where systematic distortion by refraction is less conspicuous, one finds

$$\varepsilon_{rm}(h_1 - h_2) = 0''.34$$

or, giving the concerned quantities equal weights,

$$\varepsilon_{rm}(h) = \frac{1}{\sqrt{2}} \varepsilon_{rm}(h_1 - h_2) = 0''.24.$$

The subscripts indicate that the figure in question includes the errors arising both from refraction (r) and from the very measurements of the radii (m). The last error may be derived from a similar comparison of two contours derived from one and the same crescent; evidently, the error obtained from a procedure like this will include, at the same time, the error which results from a not quite perfect shape of the preliminary centre ("centering error"). Actually, from plate No. 8 and using the above method, the mean error in one determination of height was found to be

$$\varepsilon_m(h) = 0''.16.$$

The mean error $\varepsilon_r(h)$, as produced by refraction only, is then obtained from the formula

$$\varepsilon_{rm}^2(h) = \varepsilon_r^2(h) + \varepsilon_m^2(h)$$

from which

$$\varepsilon_r(h) = 0''.18.$$

Thus, accidental refraction gives rise to an error being about equal to that produced by the measurements themselves. However, as stated above, a more serious effect of atmospheric refraction consists in systematical shifts of level which may take place within rather extended intervals of the lunar contour.

(b) A glance of the Figs. 11 and 12 reveals that HAYN's contour and the measured contours agree with regard to the general shape of the limb so that, as a rule, the various details (hills, valleys) may be recognized from one profile to the other. However, since the measured profiles themselves show up considerable differences in level, it seems

not worth while to make any numerical estimate of the agreement between the two kinds of profiles.

The topographic data gathered by HAYN is based on the investigation of a great many records of the limb. One may conclude, therefore, that the contour derived on the basis of this comprehensive material, will be rather free from influences caused by atmospheric turbulence. — A survey of the accuracies obtained from different procedures at the determination of the lunar marginal zone is given on p. 82.

CHAPTER IV

Introduction to the spectrophotometric procedure

Approach to the topic

29. *The flash spectrum records.* In Figs. 13 and 14 are reproduced some of the flash spectrum records which were obtained by LINDBLAD at the actual eclipse. The exposures are arranged in chronological order and represent, as a rule, every sixth image of the cinematographic recording, corresponding to a time interval of about $0^s.30$; the only exceptions are the upper left and the lower right exposure in Fig. 13 and the lower right in Fig. 14. It follows, since the relative speed between the Sun and Moon amounts to some $\frac{1}{2}'' \text{ sec}^{-1}$, that in all other cases the lunar edge has advanced some $0''.15$ in the meantime.

A cursory study of the behaviour of the continuous spectrum reveals at once two points of particular interest in the present context: (1) the extremely rapid change in intensity with time, (2) the strong influences from the lunar profile as visualized in the distribution of light along the lunar periphery. Both these facts display clearly that the decrease of light at the photospheric border must be very steep indeed. It is seen also that studies of the photospheric intensity are rendered possible along a fairly broad portion of the lunar limb; accordingly, the possibility is at hand to reduce or even eliminate the effects which are connected with the shape of the Moon's limb. It is natural, on the basis of these facts, to assume with LINDBLAD that a photometry of the continuous spectrum might form a useful aid for the solution of the contact problem.

Turning to the flash lines in proper sense, one recognizes the strength of these to be likewise strongly dependent on the lunar topography. This is explained by the fact that the angular depth of the reversing layer in the solar atmosphere is about the same as the angular average amplitude of the irregularities in question. With the dis-

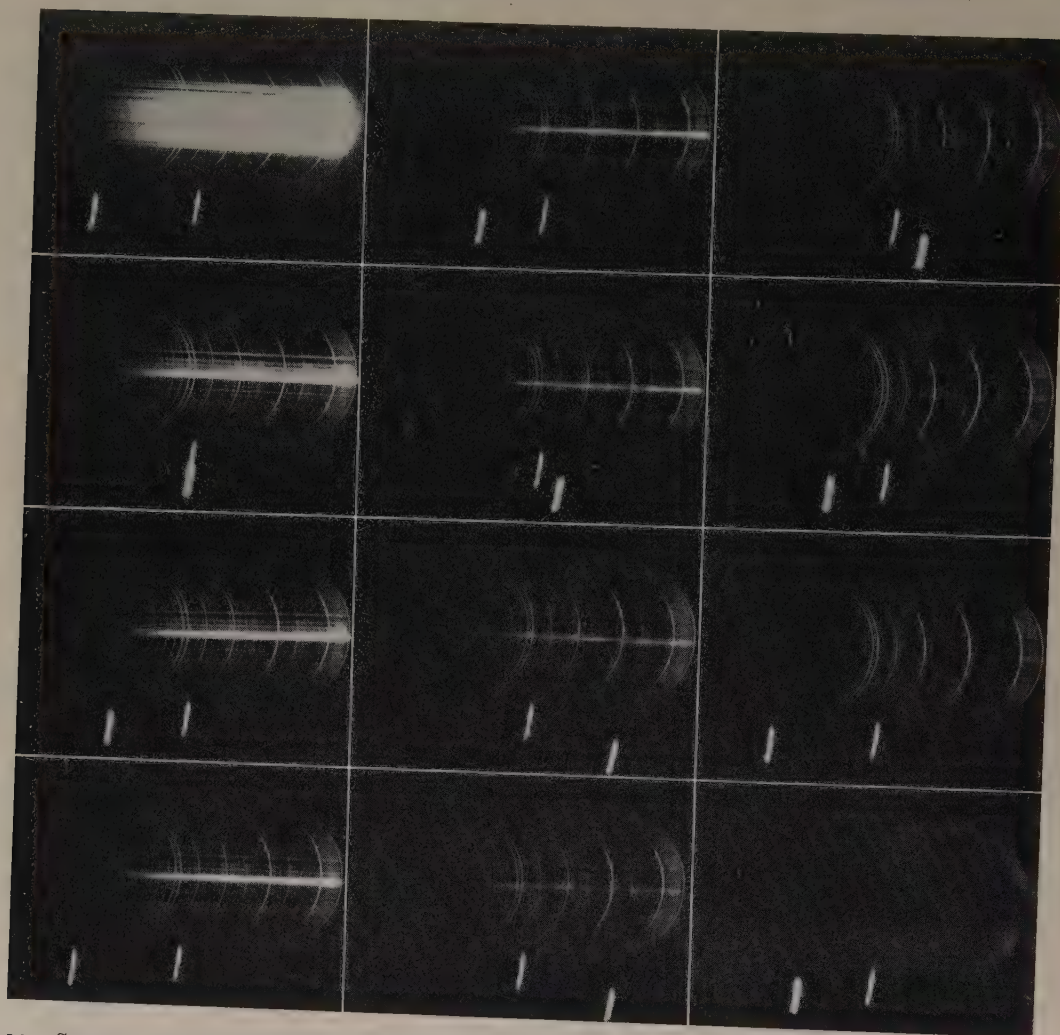


Fig. 13. Some of the flash spectra recorded by LINDBLAD at the second contact. The upper left record is taken some few seconds before the following (that below), the lower right is a record of the total phase; otherwise the interval between two consecutive exposures is constant ($0^s.30$). The two bright marks, (one fixed, the other oscillating) serve timing purposes.

persion actually employed, the great many faint lines merge into each other; the result is a formation at deep valleys of nearly unbroken stripes of light which may resemble rather greatly those being of a purely photospheric origin. Some regions, however, are distinguished by the presence of extremely faint flash lines only. In contrast to the continuous spectrum, the strongest flash lines apparently do not undergo any noticeable change in intensity, a feature which to a certain extent is to be attributed to a rather heavy exposure. The fact that the registrations were going on during the entire total phase makes it possible, however, to undertake a photometric study of the intensity gradients in the upper chromosphere even in these lines.

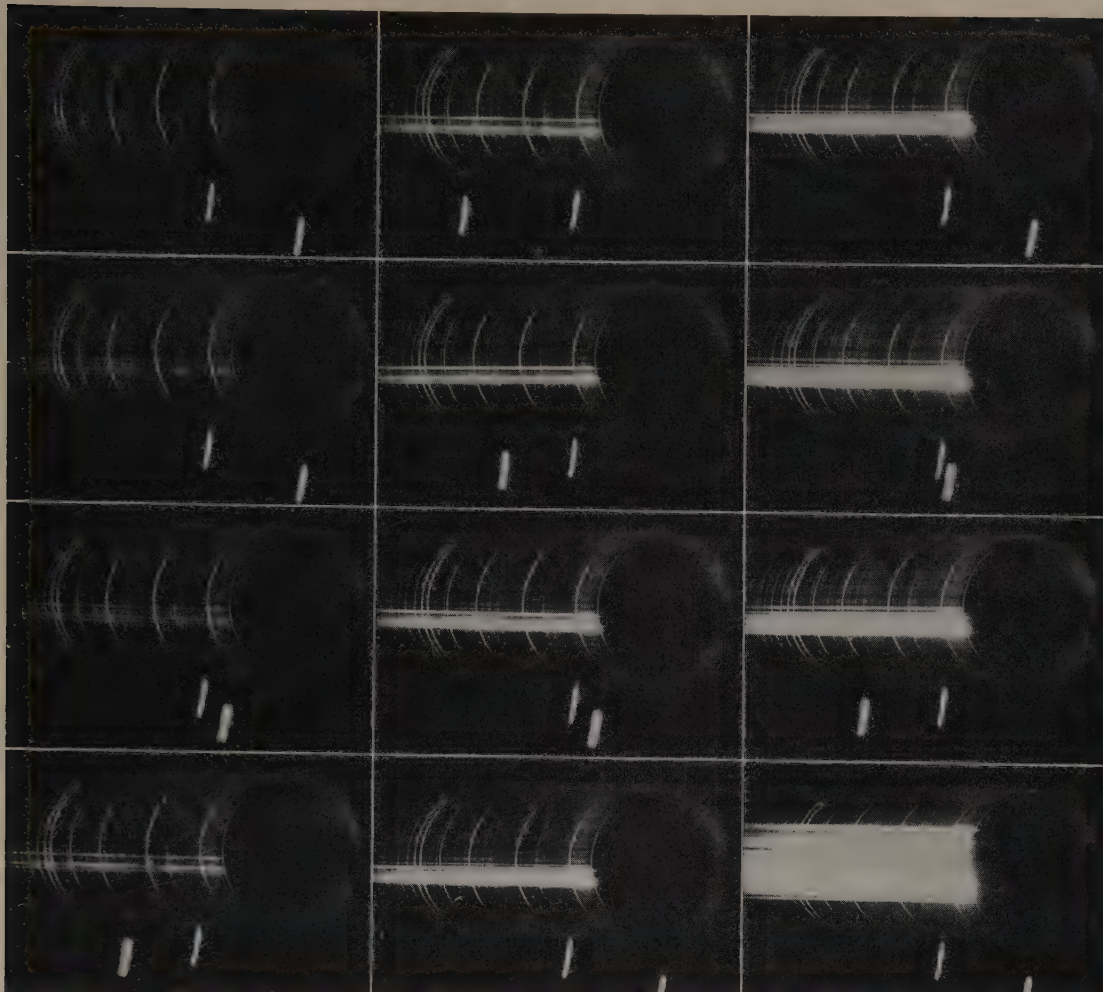


Fig. 14. Some of the flash spectra recorded by LINDBLAD at the third contact. The lower right record is taken some 2^s after the foregoing.

The flash spectra are bordered on each side by coronal stripes originating from those parts of the inner corona where the direction of dispersion is tangent to the periphery of the Moon. The coronal continuous spectrum is also to be discerned in between the lines of H_{α} and D_{β} . The original spectral records display, besides, rather distinctly a diffuse ring which is due to the green coronal line λ 5303.

30. *Line photometry versus photometry of the continuous spectrum.* Before deciding to restrict the photometric measurements for contact purposes to the photospheric spectrum only, the question should be briefly touched upon whether any advantage might be achieved by using some spectral line instead.

Obviously, not much would be gained from a procedure like that. A disappearing flash line represents, in fact, nothing else than a monochromatic record of Bailey's beads.

Therefore, when subjecting a spectral line to photometric measurements, the same difficulties would appear as when attempting a similar investigation on the beads recorded in ordinary light.

The decrease of light in the extreme photosphere is more rapid than in the solar chromosphere which makes the continuous spectrum more suitable for contact purposes. Additional excitation, e.g. in the form of low prominences, may occur in the chromosphere and thus falsify the lunar profile derived from a spectral line; the continuous spectrum produced by such excitation is, on the other hand, too weak to affect noticeably the results obtained from the photospheric spectrum.

Last but not least, as will soon be shown, a photometric research into the continuous spectrum may be achieved in a far more simple and rapid manner.

31. *Data available from a thorough photometric treatment of the continuous spectrum.* Evidently, for a given exposure and a given wavelength, the intensity of light and the distribution of light in the flash spectrum should be determined by the following data (Fig. 15):

- (a) the radial distribution of light, $i(h)$, in the concerned wavelength at the outermost solar limb,
- (b) the shape of the lunar limb,
- (c) the distance d between the solar and lunar centra, which is related to the width c of the solar crescent through $d = c + \Delta$, where $\Delta = r_{\odot} - r_{\ominus}$,
- (d) the difference in apparent radius for the Sun and Moon, Δ , which for a given value of c determines the crescent's angular breadth $2\vartheta = 2 \arccos \frac{\Delta}{\Delta + c}$ (approximately).

A complete treatment of the photometric material should accordingly render possible a determination of these four "unknowns". Of these, (a) refers to a physical property of the Sun itself whereas the remaining three merely relate to geometrical properties of the two celestial bodies. The contact problem is obviously embodied in (c), the determination of which, however, presupposes the knowledge of (b).

The function $i(h)$ may be derived independently of the other data (p. 47). This function, as we will see, constitutes the main tool of the photometric procedure for timing a total solar eclipse.

The photometry of the flash spectrum records

32. *Colour regions adapted to photometry.* The photometric measurements when carried out in the continuous spectrum must, as far as possible, be free from influences caused by the presence of spectral lines or by the presence of an appreciable fog, due, for instance, to the corona.

Actually, LINDBLAD made use of the colour region which is situated just on the short wave side of H_{β} where no single strong or even moderately strong flash line occurs right down to $\lambda 4629$ (Fe^+). The author carried out the measurements close to H_{β} , on the long wave side of this line. These two regions seem to be the most suitable ones though, no doubt, others might also be used for the same purpose.

33. *Rectilinear photometry versus photometry along the spectral curvature.* To keep the slit of the photometer when moved across the continuous spectrum within one and the same wavelength, the curvature determined by the circular lunar edge should be followed. If moved along a straight line perpendicular to the direction of dispersion the slit will evidently pass over slightly differing colour regions. The first procedure had been tried by the author, the second by LINDBLAD. We are going to see now that the latter alternative is better adapted for the actual problem of contact determinations.

To carry out the photometric investigation along the curvature of the spectrum a mechanical device, designed by Mr. N. HANSSON, was used. The spectral record was made rotatable about a centre, corresponding to the wavelength selected for investigation. The centre was marked out in the emulsion by means of a sharp needle and then the film square squeezed between two circular plates forming together a wheel; this wheel was made to rotate slowly under the friction produced against the moving horizontal border of the photometer's plateholder. The photometer, of the Moll selfrecording type, was kindly put at the author's disposal by the director of the Lund Physical Institute, Professor B. EDLÉN.

This arrangement, which so far worked very well, made possible an investigation of the light distribution in a number of wavelengths, of which some were situated between strong neighbouring flash lines. The results from the wavelength $\lambda 4100$ (the others were $\lambda\lambda 3820, 3950, 4820$ and 5775) were then utilized for the purpose of determining the moments of contacts. However, from a comparison of the results thus derived with those obtained by LINDBLAD in the H_{β} -region, it appeared that part of the discrepancies could be attributed to the restricted possibility of reproducing, at the procedure followed

by me, the settings in wavelengths from one exposure to the next. A slightly erroneous position of the indicated centre, which may likely occur as a result of the small scale (lunar radius = 3.00 mm), causes the slit to move along somewhat differing paths on the successive records. The desirable unique correspondence between angular scale and linear scale of the photometric tracing may thereby be endangered.

Even if a more perfect arrangement were to be constructed for guiding the slit along a stated circular path, this would, in fact, not give more reliable results than does a photometry carried out straight across the spectrum. For, by this procedure, the settings in the spectrum can be made in a very accurate manner; moreover, the small change of wavelength — caused by the fact that the slit moves along the tangent to the true circular path — may readily be accounted for (p. 54). Finally, rectilinear photometry is a much more timesaving undertaking than the alternative procedure.

The experiences thus gained prompted a renewed photometry of the Lund flash spectrum material according to the method used by LINDBLAD. This undertaking proved to be useful also for the reason that it offered the opportunity of bringing the treatments on the whole in better harmony with each other.

The account of the following section refers, so far as the Lund material is concerned, to the re-investigation just mentioned.

34. *Details about the rectilinear photometry.* The flash spectra have been investigated by means of the Koch-Goos selfrecording photometer of the Stockholm Observatory.

To keep the measurements within the same region of wavelength, a drawing of some strong neighbouring spectral lines was made on the screen at the diaphragm of the photometer. The projected images of the successive records were then brought to coincidence with this pattern. Since the projected image was twenty times larger than the original picture, these settings could be achieved with an accuracy of some hundredths of a millimeter, which with the dispersion used corresponds to some few units of an angstrom. Owing to the very slow gradient of density along the direction of dispersion, the slit (diaphragm) could be made rather long and, accordingly, relatively narrow. Actually, the width of the slit corresponded to an interval of 0.35 at the lunar periphery.

The obtained photometric tracings must be provided with some marks of orientation which make it possible to relate the successive tracings uniquely to each other and to the very lunar limb. LINDBLAD achieved this in extending the registrations to include the coronal stripes on both sides of the spectrum; these appear as sharp peaks in the photogram and are therefore well adapted for this purpose. The present author used

instead some small prominences, attached to the line H_{δ} , as marks of orientation. When during the registration such a small prominence passed a certain point on the screen, the shutter of the cassette containing the registering photographic paper was operated to give a corresponding mark on the photogram. This arrangement was caused by the fact, that due to the relatively smaller scale of the Lund material, the photometric tracings were realized in an essentially greater scale (40 times) than was actually used by LINDBLAD (6 times). In consequence of this, the available space on the registering paper would not have allowed the photometry to continue right out to the coronal stripes.

The cinematographic records subjected to photometry embrace a time interval of some three to four seconds at each contact. The investigation started with the exposure where only faint traces of continuous light are discernible and ended where the strong light began to fog the exposures appreciably. Between these limits, every fourth exposure has been investigated of the Stockholm material and every second to fifth of the Lund material.

Position and speed of the Moon relative to the Sun

35. *The relative speed v between the Moon and the Sun* will be needed for the further treatment of the investigated flash spectrum material. This speed has been obtained in computing, on the basis of rigid parallax formulae, the topocentric coordinates for both celestial bodies and by differentiating their differences with respect to time. The relative topocentric coordinates will, moreover, find use when determining the empirical corrections to the lunar tabular position (p. 77).

Table 4

ϱ = geocentric distance in terms of the Earth's equatorial radius, φ' = geocentric latitude, $\pi_{\odot}$ and π_{C} horizontal parallax of the Sun and the Moon, respectively

	Lund exp.	Stockholm exp.	N o t e s
Long.	21° 12' 646 E. Gr.	21° 31' 500 E. Gr.	Coord. determined by the Office of the Swedish Geographical Survey
Lat.	64° 41' 459 N.	64° 27' 077 N.	
$\varrho \sin \varphi'$	0.9004146	0.8986158	Compression of the Earth assumed to be 1/297
$\varrho \cos \varphi'$	0.4286793	0.4324634	
$\pi_{\odot}$	8".66		Parallax at mean dist. = 8".79
π_{C}	58' 8".14 - 0".02457 t		Parallax at mean dist. = 57' 02".70 (Brown), t in minutes of time from 14 ^h U.T.

The determination of the topocentric coordinates has been carried out at every fourth minute of time within an interval of 64 minutes, centered at 14^h U.T. (approximate moment of central eclipse at both stations). The basic data hereby used are collected in Table 4.

The apparent geocentric coordinates have been obtained from *The Nautical Almanac* for the year 1945. The results of the computations are given below in the form of series of powers of t , where t denotes the time in minutes from 14^h U.T. In each case, the constant term is made up of two parts, of which the first denotes the difference in geocentric coordinates at $t = 0$, whereas the second represents the relative parallax at the same moment. The latter figure has, for future use (p. 77), been determined with an accuracy of 0".001 in right ascension and 0".01 in declination.

$$\text{Lund exp.} \quad \begin{cases} \Delta\alpha \equiv \alpha'_{\odot} - \alpha'_{\odot} = + 20' 12''.7 - 20' 41''.263 + 30''.329 t + 0''.0107 t^2 \\ \Delta\delta \equiv \delta'_{\odot} - \delta'_{\odot} = + 42' 8''.6 - 42' 10''.86 - 2''.805 t - 0''.0044 t^2 \end{cases}$$

$$\text{Stockholm exp.} \quad \begin{cases} \Delta\alpha \equiv \alpha'_{\odot} - \alpha'_{\odot} = + 20' 12''.7 - 20' 58''.056 + 30''.321 t + 0''.0109 t^2 \\ \Delta\delta \equiv \delta'_{\odot} - \delta'_{\odot} = + 42' 8''.6 - 42' 4''.14 - 2''.831 t - 0''.0045 t^2 \end{cases}$$

Putting now $v_{\alpha} = \frac{d}{dt} (\Delta\alpha \cos \delta_{\odot})$ and $v_{\delta} = \frac{d}{dt} (\Delta\delta)$, we get the relative velocity from

$$v = (v_{\alpha}^2 + v_{\delta}^2)^{\frac{1}{2}}$$

$$\phi = \arctg \frac{v_{\alpha}}{v_{\delta}}$$

where ϕ is the position angle of the velocity vector and $\delta_{\odot}$ the apparent declination of the solar centre ($22^{\circ}22'.09$). Inserting the values of $\Delta\alpha$ and $\Delta\delta$ one obtains the figures of Table 5, which relate to the predicted moments t_0 of central eclipse. The columns, headed Δv and $\Delta\phi$ give the changes in v and ϕ per minute of time.

Table 5

	t_0	v	ϕ	Δv (min ⁻¹)	$\Delta\phi$ (min ⁻¹)
Lund exp.	1 ^m .0	0".4701 sec ⁻¹	95°.73	0".00034 sec ⁻¹	0°.013
Stockholm exp.	1.6	.4703	95.80	.00035	.013

Actually, the changes mentioned are so small that we, in what follows, shall adopt the given values of v and ϕ to be valid throughout the total phase; the duration of the latter amounted to only about one minute of time, implying that the true v -values differ from the adopted value by but 2 units in the last decimal at each of the two contacts.

southern or northern borders of the total belt. A non-central position will also involve a slight rotation of the crescent in position angle, which is readily verified to amount to $(v/d) \sin SMM'$ radians $\cdot \text{sec}^{-1}$ at the inner contacts (about 0.6 sec^{-1} for the Lund expedition).

The sharpness of the solar limb

36. *Introduction.* It is a well known fact that direct determinations of the darkening towards the limb of the Sun become more and more unreliable when approaching the very limb. The reason is that the scattering of rays (and the turbulence) in the terrestrial atmosphere and in the instrument itself both tend to level the true curve of energy.

To avoid this, W. H. JULIUS [14] proposed measuring the total light which is emitted from the uncovered crescent at a solar eclipse. From the rate of change of intensity, the relative emission per unit area along a solar radius may then be derived. The result is independent of atmospherical scattering and turbulence apart from unessential accidental variations due to scintillation.

The original method of JULIUS is, however, not particularly suitable for a determination of the abrupt decrease of intensity at the Sun's very border, since, in this region, the registered total intensity will be strongly influenced by the topography of the lunar marginal zone.¹ To account for this, the acting lunar contour must be known very accurately or rendered harmless in some other way. Moreover, since the phenomenon to be studied is a very rapid one (of duration some second of time), the registering instrument must have a very short time response.

The correct way of attacking the problem from the experimental point of view was shown by K. SCHWARZSCHILD [30]. At the total solar eclipse on Aug. 30, 1905, by means of a prism camera of simple construction, he secured 16 spectral records during the time interval -20° to $+10^{\circ}$ ($0^{\circ} =$ moment of second contact). The limb darkening could then be studied by following the variation of intensity at one and the same point of the lunar limb, a procedure which obviously eliminates the topographic effects appearing when studying the variation of the crescent's total light. It is clear, however, that the present problem, which bears upon the sharpness of the very limb, could not be solved on the basis of this material since, on the average, only one record was obtained per every other second.

¹ The results from earlier investigations, carried out according to the method of JULIUS, have been re-examined, amongst others, by M. MINNAERT [23], with a view of deriving the most probable shape of the solar limb.

It is obvious that a timed cinematographic recording of the flash spectrum will fulfil all conditions demanded for a detailed study of the most interesting interval of the solar radius. The exact timing of the spectral records gives a means at hand to convert the primary time scale into a height scale in seconds of arc, which is necessary for the derivation of the steep gradient of intensity and which, moreover, proves useful for a careful investigation of the lunar limb topography.

The sharpness of the solar limb has been subjected to theoretical investigations by A. UNSÖLD [32], G. RIGHINI [28] and R. WILDT [34]. These investigations differ however as to the basic assumption concerning the chemical constitution of and the opacity in the solar atmosphere. The modern idea, suggested by R. WILDT, of the opacity as originating mostly from the ionization continuum of the negative hydrogen ion has been taken into account in his computation, which may therefore be regarded as the most up to date research into the problem in question. Anyhow, the experimental establishing of the intensity gradient is one way of deciding between the different alternatives which, as stated before (p. 8), was one of the principal aims of the present cinematographic registrations.

37. *The photospheric gradient at $\lambda 4100$ as derived from the present material.* The decrease of light, within the portion of the solar radius covered by the present photometry, has been obtained by studying the rate of change of intensity at six selected points in the continuous spectrum. Three of these points refer to the eastern limb (second contact) and the remaining three to the western limb (third contact). They were chosen so as to correspond to different details of the lunar contour, e.g. hills, valleys, etc. Before entering upon the results derived [(b) below], we give some words about the calibration of the photometric measurements.

(a) *The characteristic curve* for the film emulsion has been obtained with the aid of a tungsten lamp, for which the radiation temperature (T_s) at $\lambda 6530$ was known as a function of the strength of the heating current (I). The lamp together with its calibration data was kindly put at our disposal by Professor B. EDLÉN.

The continuous spectrum of the tungsten lamp was recorded by means of the prism camera of the Lund expedition, now used in conjunction with a slit and a collimating lens. The different steps of intensity were realized in changing the I -values, step by step, by 0.25 amp.; in doing so, the time of exposure was kept constant by using as shutter a uniformly rotating wheel with a transparent sector. Two series of calibration exposures were obtained, one for increasing, the other for decreasing values of the heating current.

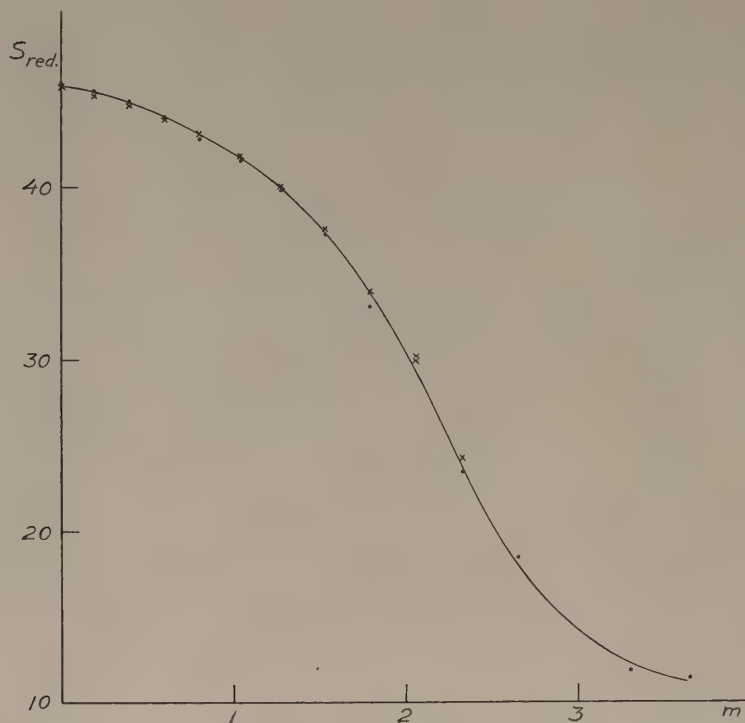


Fig. 16. The characteristic curve for the region λ 4100. Crosses and dots denote points obtained for increasing and decreasing values of the heating current, respectively.

The relation between radiation temperature, T_s , and true temperature, T , is

$$E_{\lambda T_s} = e_{\lambda T} \cdot E_{\lambda T}$$

where $E_{\lambda T}$ is the Planckian function and $e_{\lambda T}$ the spectral emissivity of tungsten, the latter thoroughly investigated by H. C. HAMAKER [9]. Now, for the actual moderate temperatures ($< 2500^\circ$ K), the Planckian law may be substituted by that of Wien ($E_{\lambda T} \sim \lambda^{-5} e^{-c/\lambda T}$), so that using this and taking the natural logarithm ($\ln$) of both members one can rewrite the foregoing expression in the form

$$\frac{1}{T} - \frac{1}{T_s} = \frac{\lambda_0}{c} \ln e_{\lambda_0 T}.$$

Putting here $\lambda_0 = 6530 \cdot 10^{-8}$ cm, $c = 1.432$ cm · degree, one gets the true temperature T from T_s by means of successive approximation in two steps (first approximation $e_{\lambda_0 T} = e_{\lambda_0 T_s}$). The relation between the two quantities was found to be closely approximated by

$$T = 1.163 T_s - 160^\circ$$

which holds within the interval $1500^\circ < T < 2500^\circ$. The monochromatic intensity, for

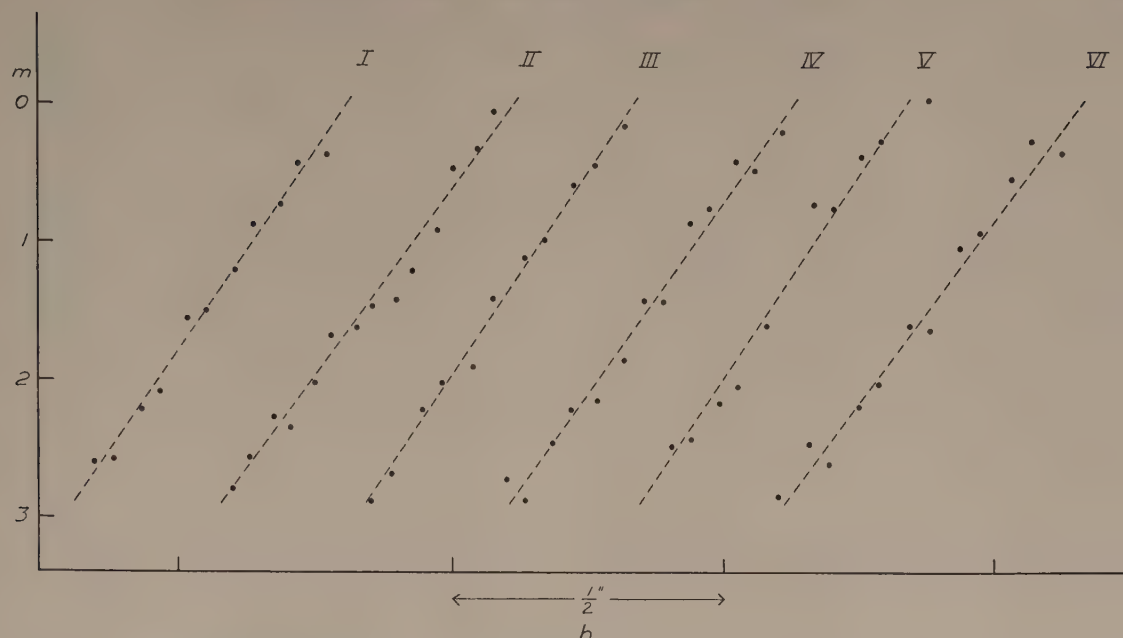


Fig. 17. Monochromatic gradients ($\lambda 4100$) determined in six points at the Sun's edge. Horizontal scale gives relative height in the photosphere (scale value given below). Vertical scale stellar magnitudes.

successive values of I (or T_s) and for an arbitrary wavelength λ , may now be obtained directly from the product $e_{\lambda T} \cdot E_{\lambda T}$. The procedure in question was followed primarily because of the fact that it rendered possible an examination of the γ -values all over the spectrum. The characteristic curve as obtained in the region $\lambda 4100$ is shown in Fig. 16.

The exposure time employed at the calibration measurements was about 50 times longer than that of the original cinematography. The time factor in question has been considered as negligible with regard to its influence on the characteristic curve. The same matter was studied in greater detail by LINDBLAD, when treating the material of the Stockholm expedition. By using as a light source in the tube sensitometer commercial flash bulbs (Philip Photoflux, Type 2), he was able to reduce the exposure time to nearly the same as that of the cinematographic registrations. For a time factor of some two hundred he found in this way the scale corrections to be of an almost insignificant importance.

(b) *The results.* For each of the six points in the continuous spectrum, the obtained magnitudes m when plotted against the moments of exposure were found to group with small scattering about a straight line. From the slope of this line, $\frac{dm}{dt}$, the gradient of intensity $\frac{dm}{dh}$ along a vertical in the photosphere is obtained from the identity

$$\frac{dm}{dt} = \frac{dm}{dh} \cdot \frac{dh}{dt}$$

or, putting $\frac{dh}{dt} = v_p$ (p. 45)

$$\frac{dm}{dh} = \frac{1}{v_p} \cdot \frac{dm}{dt}.$$

The magnitude m refers to the total (cumulative) intensity of a luminous photospheric strip, the length (height) of which is determined by the actual position of the lunar limb whereas its breadth may be taken equal to the width dx of the acting slit of the photometer (Fig. 15).

It is seen from Fig. 17 that, in each of the six cases, the individual points scatter moderately around a straight line, indicating that the relation $m(h)$ is linear within the

Table 6

Determination of the decrease of light at $\lambda 4100$ at the Sun's outermost limb

Point No.	Pos. angle	Limb feature	dm/dh per arc second
I.....	68.9	hill	$5^m.87 \pm 0^m.22$
II.....	58.0	slope	5.45
III.....	88.4	valley	5.92
IV.....	304.5	valley	5.60
V.....	299.5	plane	5.94
VI.....	287.9	slope	5.33 ± 0.26
			Mean 5.69 ± 0.11

concerned interval of the argument. The different gradients dm/dh as determined by the method of least squares do not differ more from their mean value than might be expected from the internal mean error affecting the gradients themselves (Table 6). The type of the corresponding lunar object seems not to affect the quantities in question, with the possible exception for those obtained at a mountain-slope of the limb topography; it is probable, however, that this tendency is merely accidental since, if real and interpreted as caused by the finite slit width or by one or other photographic neighbourhood effect, the values obtained at a lunar valley and at a lunar hill should differ even more from each other. Therefore, we shall accept the mean

$$\frac{dm}{dh} = 5^m.69 \pm 0^m.11 \text{ per second of arc}$$

as the final value of the photospheric gradient at $\lambda 4100$, which obviously refers to the part of the solar limb situated outside the inversion point of the energy distribution.

The cumulative (integrated) intensity $I(h)$ runs consequently exponentially with h which must also be the case for the true intensity $i(h)$; this follows from the fact that the latter is simply obtained from $I(h)$ by differentiation with respect to h . Hence, one obtains the relative decrease of light at $\lambda 4100$ to be

$$\begin{aligned} i(h) &= \text{const. } 10^{-0.4 \cdot 5.69 h} \\ &= \text{const. } 10^{-2.28 h} \end{aligned}$$

where h is to be reckoned in seconds of arc.

To get the shape of the entire solar limb, the measurable interval of intensity has to be increased. This may be achieved by using simultaneously working instruments of different light gathering powers, or by providing one single instrument with some arrangement which makes it possible to diminish, at the proper moment, the effective focal ratio (filter, adjustable diaphragm). The Lund expedition to Brazil [38 p.

79] in 1947 employed for the same purpose as camera shutter a rotating disk with two opposite transparent sectors of different angles; thus, alternate long and short exposures could be obtained which, moreover, were chosen so as to have the registered intensities overlapping.

38. *Comparison with the results of LINDBLAD and WILDT.* As stated before, LINDBLAD carried out the photometry in the wavelength region $\lambda 4800$. The gradient derived by him on the basis of these measurements was found to be

$$\frac{dm}{dh} = 4^m.86 \text{ per second of arc.}$$

The result thus obtained turned out to be in agreement with the distribution of continuous radiation argued by WILDT [34]; the agreement is shown by the two curves in Fig. 18. The present value, $5^m.69$ per second of arc, is somewhat greater than

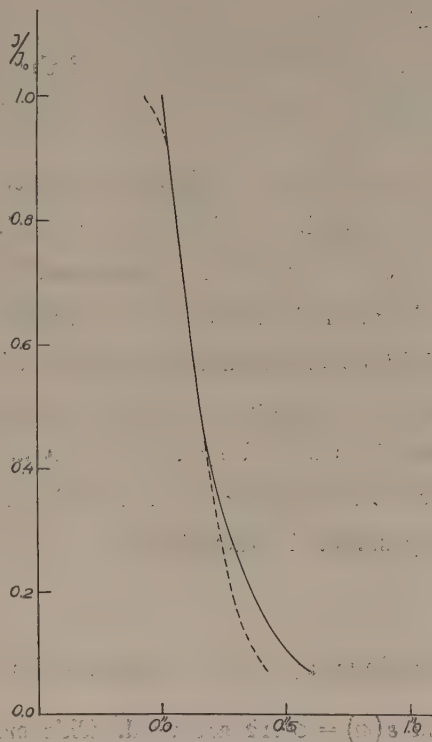


Fig. 18. The empirical gradient as derived by LINDBLAD at $\lambda 4800$ (full drawn) and the gradient of the continuous radiation as calculated by R. WILDT.

¹ LINDBLAD's results concerning his determinations of the gradients at $\lambda 4800$ and the Balmer limit are so far unpublished, except for a brief communication to WILDT, quoted by the latter in ApJ 105, p. 36.

that of LINDBLAD. Yet, it is likewise in strong favour of WILDT's gradient so that, as a matter of fact, both of the empiric investigations support the assumption of the negative hydrogen ion as responsible for the main part of the solar photospheric opacity.

39. *Estimate of the mean error in one photometric determination of height.* The mean error affecting a single determination of magnitude (as found from the residuals $m - m_c$, where m_c is the value corresponding to the linear relation $m(h)$) amounts to about $0^m.12$. This error is somewhat too high to result from the very photometric procedure which might be expected to contribute with about $0^m.04$ only. It seems natural to explain the actual scattering as resulting from light variations due to atmospheric scintillation¹; vibrations caused by the camera motor may also contribute towards this result. Anyhow, the corresponding error $\varepsilon(h)$ in a photometric height determination turns out to be very small. In fact, from

$$\varepsilon(h) = \frac{dh}{dm} \cdot \varepsilon(m)$$

one finds $\varepsilon(h)$ to be of the order $0''.02$ when the values $\frac{dm}{dh} = 5^m.69$ per second of arc and $\varepsilon(m) = 0^m.12$ are used. This error is far less than that obtained from direct measurements on ordinary photographic records, which has previously been stated to amount to more than $0''.2$ (p. 36).

CHAPTER V

The solution of the contact problem on the basis of the spectrophotometric procedure

It has been shown in the preceding chapter that a cinematographic spectral recording of the inner contacts at a total solar eclipse renders possible a determination of the rapid decrease of intensity at the very edge of the Sun. The relation $m(h)$ between the cumulative magnitude m and the height h in the photosphere has been found to be linear. We are now going to show that the inverse function $h(m)$ is a very convenient tool when dealing with the problem of photometric contact determinations.

40. *The establishment of a sharp solar edge.* So far, the darkening function $m(h)$ has been studied without stating the zero point of the argument. For the present pur-

¹ M. MINNAERT and J. HOUTGAST [24], from a photometry of star trails on a photographic plate, found for Capella at a similar altitude (36°) and a comparable aperture of the instrument the corresponding figure to be about $0^m.08$. Probably, however, scintillation will be more pronounced during day-time which renders the above figure $0^m.12$ reasonable.

pose, however, it will be found suitable to consider h to be zero for a certain value m_0 so that, by definition,

$$h(m_0) = 0.$$

The geometrical correspondence to this condition is an isophote running along the solar periphery. For the sake of simplicity, we shall regard m_0 as corresponding to the highest intensity (density) actually registered at the photometric investigation of the flash spectra; in doing so, the photometric heights $h(m)$ will all have positive sign.

The isophote in question constitutes the photospheric zero level with respect to which the position of the irregular lunar edge will be determined. This artificial solar edge has the advantage of being sharp and well defined, in contrast to the more or less diffuse edge of a photographic solar record, taken in ordinary light. Since the decrease of light at the extreme limb follows the same law around the whole solar periphery, the zero level must have the same geometrical shape as the apparent border of the Sun's disc. Amongst others, F. HAYN [11] has found that no difference can safely be stated between the polar and equatorial radii of the Sun. We are therefore entitled to assume the adopted zero level to be of a circular form, all the more as we will be dealing with but a limited portion of the solar periphery at each contact.

If we establish photometrically the position of the lunar edge relative to the photospheric zero level, the measurements will of course be entirely differential. A consequence of this is that differential refraction and instrumental distortion of the image will not affect the results here. We have seen already that the mean error in one determination of h , effected in this way, is very small, about 0".02 (p. 52).

41. *The practical performance of the height determinations.* It is evidently not necessary to consider the individual m -values explicitly. As a matter of fact, the unique correspondence between the spectral intensity and the density S (as given by the characteristic curve $m(S)$) makes it reasonable to use instead the primary S -values as an indicator of position of the lunar edge. This is readily effected by combining the relations $m(S)$ and $h(m)$ into the desired function $h(S)$. It remains only to fix the S -value corresponding to the level $h = 0$.

Table 7 gives the connection between S and h derived in this way. The figures are based on the gradient 5^m.69 per second of arc (p. 50) and the characteristic curve reproduced in Fig. 16. It should be pointed out that even if the gradient used is not quite correct — due for instance to some error in the photometric scale — the relation $h(S)$ will still hold since the intensity is employed only intermediately. It is clear, moreover, that the same connection might have been obtained directly from the primary

Table 7

S denotes reduced plate density. Unit of height h : one hundredth of a second of arc.

S	h	S	h
45.9	00	36.8-37.4	27
45.7-45.8	01	36.2-36.7	28
45.6-45.6	02	35.4-36.1	29
45.5-45.5	03		
45.3-45.4	04	34.7-35.3	30
45.1-45.2	05	33.9-34.6	31
44.9-45.0	06	33.0-33.8	32
44.7-44.8	07	32.1-32.9	33
44.5-44.6	08	31.1-32.0	34
44.3-44.4	09	30.0-31.0	35
		29.0-29.9	36
44.1-44.2	10	27.8-28.9	37
43.8-44.0	11	26.6-27.7	38
43.6-43.7	12	25.4-26.5	39
43.3-43.5	13		
43.0-43.2	14	24.2-25.3	40
42.7-42.9	15	23.0-24.1	41
42.4-42.6	16	21.8-22.9	42
42.1-42.3	17	20.7-21.7	43
41.7-42.0	18	19.8-20.6	44
41.2-41.6	19	18.9-19.7	45
		18.1-18.8	46
40.8-41.1	20	17.4-18.0	47
40.3-40.7	21	16.6-17.3	48
39.8-40.2	22	16.0-16.5	49
39.3-39.7	23		
38.7-39.2	24	15.4-15.9	50
38.2-38.6	25	14.9-15.3	51
37.5-38.1	26	14.4-14.8	52

density data. However, the spectral intensity is needed when deriving some small corrections to the photometric heights obtained from Table 7 (see next section).

42. Corrections to the photometric heights.

The h -values obtained from Table 7 must, before use, be corrected for the small shift of wavelength caused by the rectilinear photometry and for the fact that the spectral magnitude m refers to the total intensity within a photospheric strip oriented parallel to the direction of dispersion (and thus generally deviating from the direction of the corresponding solar radius). It may happen also that the speed of the recording flash camera changes during the time of registration; such an event will affect the time of exposure and accordingly the position of the photospheric standard level.

All these effects may be considered as affecting primarily the observed spectral magnitude, which as a result of this may take the value $m + \Delta m$ instead of the correct value m . The apparent photometric height is accordingly

$$h(m + \Delta m) = h(m) + \Delta m \left(\frac{dh}{dm} \right)$$

which shows the correction in height to be

$$\Delta h = - \Delta m \left(\frac{dm}{dh} \right)^{-1} \quad (1)$$

(a) The reduction of the photometric heights to a standard wavelength λ is readily obtained from a glance at Fig. 19. Instead of describing a circular path of constant wavelength λ , the photometer beam now actually passes along the straight horizontal

line, perpendicular to the direction of dispersion y (positive downwards with increasing λ and passing through the centre C). The linear error in the photometer setting at a point Q corresponding to the angle θ at the centre is obviously $y = r(1 - \cos \theta)$. The apparent change in magnitude per unit displacement along the y -axis may be written (S = density)

$$\frac{dm}{dy} = \frac{dm}{dS} \cdot \frac{dS}{dy}.$$

Here, dm/dS has to be obtained from the characteristic curve and dS/dy from a photometry along the y -axis in the region concerned. The corresponding correction in height is then obtained from (1).

Table 8 gives the result as found in the region $\lambda 4100$, for three different values of the density; the characteristic curve used is that reproduced in Fig. 16.

Table 8

Units of S	dm/dS	dS/dy	dm/dy	dh/dy
23	— ^m .0078	7.9 mm ⁻¹	— ^m .062 mm ⁻¹	— ^o .011 mm ⁻¹
27	— .0085	7.3	— .062	— .011
40	— .0200	3.3	— .066	— .012
		Mean	—0.063	—0.011

The correction in height, per millimeter displacement in the spectrum, amounts to only some —^o.011 (Table 8). For $\theta = 25^\circ$ — a rough maximum value — and $r = 3.00$ mm, one finds the maximum correction for the Lund material to be some —^o.003 which is negligible.

The gradient dh/dy for LINDBLAD'S spectra was found to be even less, in accordance with the fact that the colour region used by him was situated more to the red end of the spectrum ($\lambda 4800$) where the spectral gradient of intensity is numerically less. For, obviously, when within the concerned narrow colour interval the emulsion sensibility is constant and no disturbing flash lines are present, the corrections in question must be due to the spectral gradient only; and this is a reasonable explanation of the fact that

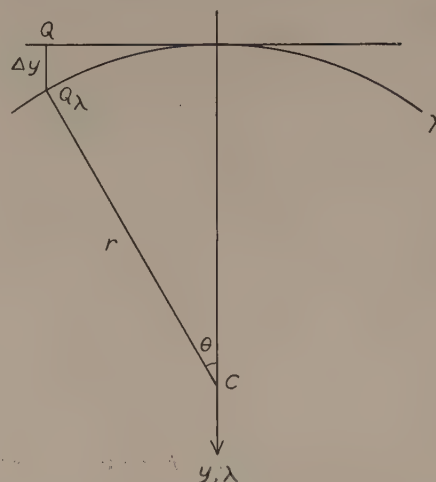


Fig. 19. The circular curve corresponds to a certain wavelength λ in the continuous spectrum, C is the centre. The wavelength correction implies a reduction of the density found at Q to the correct value in Q_λ .

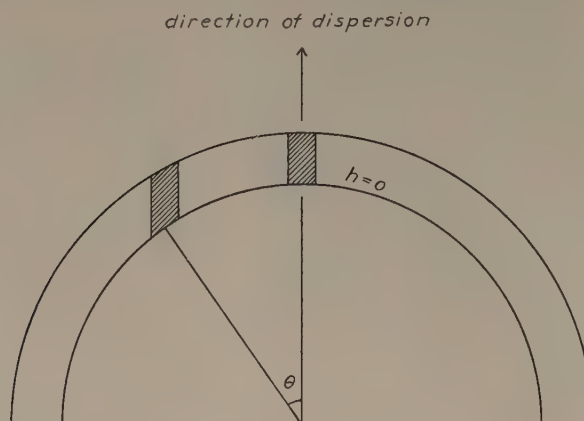


Fig. 20. The semi-circle $h=0$ represents a certain photospheric level at the Sun's limb. The summing-up of the intensity in the direction of dispersion causes the intensity per unit area in the flash spectrum to increase with the central angle θ .

the quantities dm/dy in Table 8 are nearly independent of S or m . The spectral gradient deduced from the radiation law of Wien (with $T = 4800^\circ \text{K}$ and $\lambda = 4100 \text{ \AA}$) amounts to -0.058 per $100 \text{ \AA}$; with the actual dispersion of $100 \text{ \AA}$ per millimeter at this wavelength, the agreement between the empirical and theoretical gradients turns out to be rather good.

Since the settings in wavelengths at the photometric treatment of the flash spectra could be reproduced, from one record to the next, with an accuracy of some few hundredths of a millimeter (p. 42) it follows that the error caused thereby in the photometric heights is entirely beyond any importance.

To summarize the results of the above discussion, the errors arising from the rectilinear photometry are negligible. Moreover, when using the same colour region for both photometric materials, they will affect the contact times in the same way and accordingly cancel in the differential contact problem.

(b) Reduction to radial dispersion may be achieved by means of Fig. 20 where the two semi-circles signify the zero level and the "outer" border of the solar photosphere. The two shadowed strips, both of unit breadth and parallel to the direction of dispersion, correspond to the central angles $\theta = 0$ and $\theta = \theta$, respectively. Denoting by m and $m + \Delta m$ the respective magnitudes of the total intensities, we have

$$\Delta m = -2.5 \log \sec \theta$$

since the surfaces of the strips are in the proportion $1 : \sec \theta$. The corrections Δh found from (1) are given in Table 9 for one of the expeditions.

Table 9
Corrections to radial dispersion for
 $dm/dh = 4^m.86$ per second of arc (LINDBLAD)

θ	Δh	θ	Δh
0°	$+0''000$	20°	$+0''014$
5	.001	25	.022
10	.003	30	.032
15	.008	35	.045

(c) Actually, for one of the expeditions the camera output per second of time was different at the registrations of the second and third contacts, in fact 22.08 (n_2) and 20.53 (n_3), respectively. If we assume the law of reciprocity to hold good this means that at the latter contact the recordings have obtained an additional exposure

$$\Delta m = -2^m.5 \log n_2/n_3.$$

Reduction to zero level of the second contact is consequently realized by adding to the heights derived at the third contact the amount Δh obtained from (1). With the above values of n_2 and n_3 and with $dm/dh = 4^m.86$ per second of arc (Stockholm expedition) one finds

$$\Delta h = +0''016.$$

43. *Preservation of the photospheric zero level.* Since the solar isophote $h = 0$ is used to define the contacting solar edge, it must be of great importance to keep the level unchanged, particularly from one contact to the other. On the other hand, for the geodetic project, it is not necessary at all for the two observers to employ exactly the same level because two different isophotes still define the same solar centre.

The present problem may be expressed in the following way. Let us assume the zero level to be defined from a specified set of standard instrumental and atmospherical properties. A disturbance in any one of these properties will cause a corresponding change Δh in the zero level. The task, therefore, is to derive the different quantities Δh , thereby rendering possible a reduction back to the standard conditions and to the standard zero level. It is a chief advantage of the photometric method that it is able to master this problem. Also, that — on account of the steep gradient of intensity within the region where the measurements are carried out — the disturbances Δh generally will be very small.

The sources of error just dealt with have to be searched for in the registering photometer itself, in the properties of the photographic emulsion and the registering instrument and, finally, in the terrestrial atmosphere.

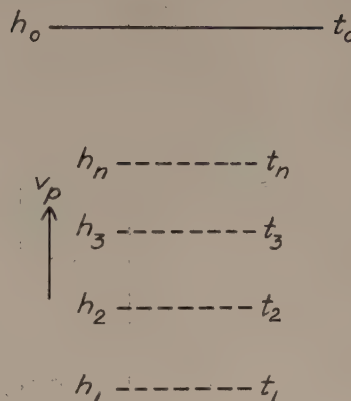
The influences arising from the photometer may not likely be of any appreciable importance, at any rate if some simple measure is taken to control, now and then, the deflections corresponding to some few standard densities. The Koch-Goos photometer of the Stockholm Observatory is equipped with an arrangement of this kind. The proceeding used is to record, at the beginning and end of every photometric registration, the deflections caused by putting into the beam of light a series of standard filters, covering a range of transparency from zero to one hundred per cent. The densities S may thereby be reduced to a standard calibration curve.

A first control of the sensibility of the photographic emulsion is already at hand in the form of prominences and in the coronal stripes on both sides of the flash spectra. Both of these should be expected to give a constant photographic impression throughout the whole time of cinematographic registration. Even better is, of course, to have some terrestrial light signal of proper colour and intensity photographed simultaneously with the flash spectra themselves. A convenient procedure, which controls not only sensibility but also gradation, is recording continuously a light beam which has passed through a rotating polarizer and a fixed double-image analyzer. This arrangement, suggested by Dr. Y. ÖHMAN, and employed by the Stockholm solar eclipse expedition to Brazil on May 20, 1947 [35] [38, p. 75], is particularly well adapted for detecting possible running changes in the emulsion properties.

Concerning the instrumental properties, we have been dealing already with how to correct for a varying time of exposure. If a parabolic mirror of Newtonian type is used, it is important to control that all light entering the instrument is reflected by the secondary; if not, a slightly varying pointing of the instrument towards the Sun will affect the light gathering power, for which reductions are not easily achieved.

The terrestrial atmosphere may exert influences on account of a varying transparency, not necessarily caused by interfering clouds only. However, in the case of a clear sky, during the short time of the total phase, the extinction cannot likely be subjected to any considerable change; if the transparency varies with, say, 5 per cent (which may be regarded as a maximum value), the influences in Δh will be 0".01, an amount still harmless for the relative contact problem. If clouds interfere, the situation will be more serious. Thin and uniformly distributed clouds or a hazy sky need not necessarily prevent getting useful results, even if such a state of things is difficult to judge satis-

factorily for a terrestrial observer. It might be that an extension of the photometric measurements so as to cover the whole solar limb, part of the extreme disk included, would be found useful in such a case; the geometrical shape of the limb should be unaffected by a changed extinction and accordingly, by an intercomparison of the results from the two contacts, render possible an independent estimate of the relative position of the Sun and the Moon.



44. *Determination of the shape and position of the lunar contour.* (a) *Formal description.* It is easy, on the basis of the photometric procedure just outlined, to derive the shape and position of the lunar contour as valid for a

definite moment t_0 . Suppose that the photometric heights h_i , corresponding to the moments t_i of the successive exposures, have been determined for a certain point of position angle p at the lunar edge. The circumstances are visualized in Fig. 21 where the lunar edge is assumed to move upwards from the photosphere with the relative velocity v_p (p. 45). Starting from the heights h_i , we are now able to derive the position h_{0i} of the lunar limb at the arbitrary moment t_0 , using the simple relation

$$h_{0i} = h_i + v_p(t_0 - t_i). \quad (2)$$

One obtains in this way as many independent statements of the reduced heights h_{0i} as there are exposures used to determine the momentary heights h_i .

The last term in the expression (2) denotes the relative displacement taking place during the involved time interval, thus a quantity known accurately from v_p and the oscillographic registration of time. The scattering of the individual h_{0i} about the mean

$$h_0 = \overline{h_{0i}}$$

must consequently result from the stated photometric heights themselves. The mean error $\varepsilon(h_0)$ will inform us about the precision of the photometric height determinations.

The reduced height h_0 , accordingly, is the perpendicular distance of the lunar limb to the photospheric level, at position angle p and at standard moment t_0 . The shape and position of the lunar contour, as referred to the same level and the same moment t_0 , is then obtained by repeating the above procedure for a convenient number of position angles p . The operation in question has, of course, to be carried out at both contacts

Fig. 21. The dashed lines indicate the momentary positions of the lunar edge referred to the adopted zero level.

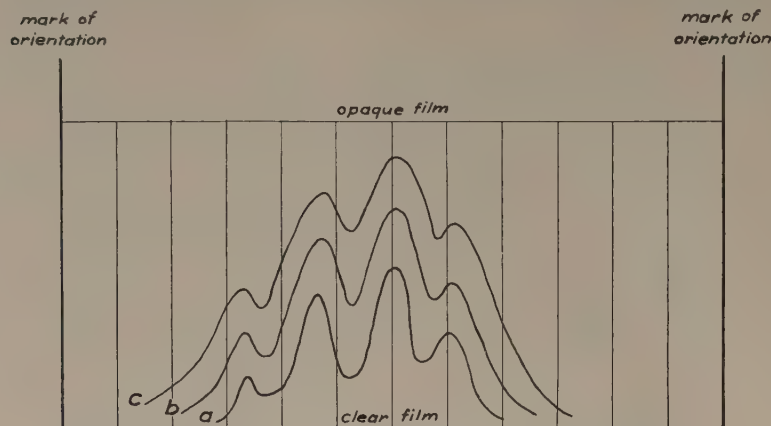


Fig. 22. Determination of the density at specified points in the photometric tracings across the flash spectrum. The density is read off at the vertical lines of the glass scale oriented by means of the orientation marks at both sides of the tracing. The letters a, b, c refer to three consecutive photometric tracings.

so that we in this way get the two portions of the lunar contour determined at two standard moments t_{02} and t_{03} .

(b) *Applications and results.* It has been related before how marks of orientation may be achieved in the photometric tracings across the flash spectra (p. 42). The marks in question are necessary to define the unique correspondence between analogous points in the successive tracings and, moreover, to define the connection between the abscissae x of the tracings and the angular scale θ along the lunar edge (Fig. 24).

The procedure of determining the lunar profile, which relates most directly to the description given in (a) above, is realized as follows (Fig. 22). A transparent glass scale provided with vertical lines of equal spacing is placed above the photometric tracing and adjusted in a definite manner with regard to the orientation marks at both ends of the recording. The densities S are read at each point of the photometric curve where this is cut by the successive lines of the glass scale. The operation is then repeated for all exposures which have been subjected to photometric investigation. One obtains in this way, by means of Table 7, the successive photometric heights h_i corresponding to definite equally spaced points at the lunar edge.

In the actual case, the investigation in question was considerably facilitated by the fact that the registrations of the Koch-Goos photometer of the Stockholm Observatory are provided with horizontal lines, parallel with the x -axis of the tracings, which render possible a direct reading of the density values. The latter, reduced to a mean calibration curve of the photometer (p. 58), have been utilized at the further treatment only when they fulfilled the condition $14.4 < S < 45.9$; the deflections, otherwise, ranged from 8 units

Table 10

p = position angle, v_p = relative displacement along the Sun's radius per second of time, h_0 = mean value of the reduced heights h_{0i} (printed in italics), $\varepsilon(h_{0i})$ = mean error in one photometric determination of height, $\varepsilon(h_0)$ = mean error in the mean. Unit of height: second of arc (cf. explanation in text p. 62).

Second contact				Third contact			
Exp. No.	$t_i - t_{02}$	I	III	Exp. No.	$t_i - t_{03}$	IV	V
396	-0 ^s .929	18.6 .46 .39 .85		70	1 ^s .341	17.1 .48 .56 1.04	
398	-1.010	19.0 .45 .42 .87		72	1.422	15.3 .51 .59 1.10	
401	-1.130	26.0 .39 .47 .86	15.3 .51 .53 1.04	75	1.544	21.0 .43 .64 1.07	
403	-1.212	28.7 .37 .51 .88	17.6 .47 .57 1.04	77	1.625	26.2 .39 .68 1.07	15.6 .50 .70 1.20
406	-1.331	37.0 .27 .56 .83	26.3 .39 .62 1.01	80	1.747	27.7 .38 .73 1.11	20.6 .44 .76 1.20
408	-1.413	37.7 .26 .59 .85	30.3 .35 .66 1.01	82	1.828	33.0 .32 .76 1.08	21.7 .43 .79 1.22
411	-1.532	40.5 .21 .64 .85	32.2 .33 .71 1.04	85	1.949	38.6 .25 .81 1.06	27.3 .38 .84 1.22
413	-1.614	42.7 .15 .68 .83	38.6 .25 .75 1.00	87	2.030	38.4 .25 .84 1.09	29.6 .36 .88 1.24
416	-1.733	43.5 .13 .73 .86	41.2 .19 .81 1.00	90	2.152	42.8 .15 .90 1.05	36.7 .28 .93 1.21
418	-1.815	44.7 .07 .76 .83	42.1 .17 .85 1.02	92	2.233	43.4 .13 .93 1.06	
421	-1.934	44.9 .06 .81 .87	44.1 .10 .90 1.00	95	2.355	44.8 .07 .98 1.05	43.5 .13 1.02 1.15
423	-2.016		44.7 .07 .94 1.01	97	2.437	44.5 .08 1.01 1.09	43.4 .13 1.06 1.19
426	-2.137		45.6 .02 1.00 1.02	100	2.560	45.5 .03 1.06 1.09	44.9 .06 1.11 1.17
				102	2.641		45.3 .04 1.14 1.18
				107	2.844		46.0 .00 1.23 1.23
p		68°9	88°4	p		304°5	299°5
v_p		"4196	"4663	v_p		"4159	"4334
h_0		"853	1"017	h_0		1"074	1"201
$\varepsilon(h_{0i})$		"017	"016	$\varepsilon(h_{0i})$		"021	"027
$\varepsilon(h_0)$		"0052	"0049	$\varepsilon(h_0)$		"0059	"0081

(clear plate) to 46.5 units (complete opacity). The spacing of the vertical lines of the transparent glass plate corresponded to $0^{\circ}.72$ in angular measure at the lunar marginal zone.

To show how, on the basis of this data, the computation of the lunar profile has been carried through in practice, the results from four different points have been gathered in Table 10 (Lund exp.). These points, numbered I, III (second contact) and IV, V (third contact), are the same as those of corresponding numbers in which the gradient of intensity at the Sun's very limb has been previously determined (Table 6, p. 50). The meaning of the notations in Table 10 are the same as before; they are moreover explained in the head of the table. It should be mentioned only that the standard moments t_{02} and t_{03} have been chosen equal to the moments of exposure of certain cinematographic records, namely records No. 373 and No. 37, respectively. The four figures belonging to a specified exposure and a specified point are arranged as in the following scheme, which also gives their significations:

$$\begin{array}{ll} S \text{ (density)} & h_i \text{ (photometric height)} \\ v_p (t_0 - t_i) & h_{0i} \text{ (reduced photom. height).} \end{array}$$

The chief interest is, of course, connected to the behaviour of the quantity h_{0i} (printed in italics) which, as stated in (a) above, is made up of the sum of h_i and the displacement $v_p(t_0 - t_i)$. We find the mean error $\varepsilon(h_{0i})$ in one photometric determination of the relative position to be approximately $0^{\circ}.02$, in agreement with the estimate made on p. 52. The mean error $\varepsilon(h_0)$, where $h_0 = \overline{h_{0i}}$, is in none of the four cases as great as $0^{\circ}.01$. This is a considerable accuracy, revealing high efficiency of the spectrophotometric procedure of contact determinations. It should be recalled that an accuracy of $0^{\circ}.01$ when obtained instead from direct measurements on large scale photographic records of the eclipse would require a linear precision of about 10^{-4} mm.; this is, no doubt, far beyond the reach of any measuring machine.

The computations demonstrated in Table 10 have, as mentioned, been carried out for intervals of p equal to $0^{\circ}.72$. If the lunar profiles thus obtained are visualized in a series of diagrams, one for each exposure, we get a picture like that in Fig. 23, where the profiles derived by LINDBLAD have been reproduced for every twelfth exposure. Actually, the procedure used by LINDBLAD was somewhat different from that outlined above. It consisted in stating the abscissae x of those points in the continuous spectrum which corresponded to selected standard values of the density and thus (apart from the small corrections treated in section 42) to standard values of photometric height h . The photometric contour is thereby obtained as the locus of the point (x, h) ; in doing so,

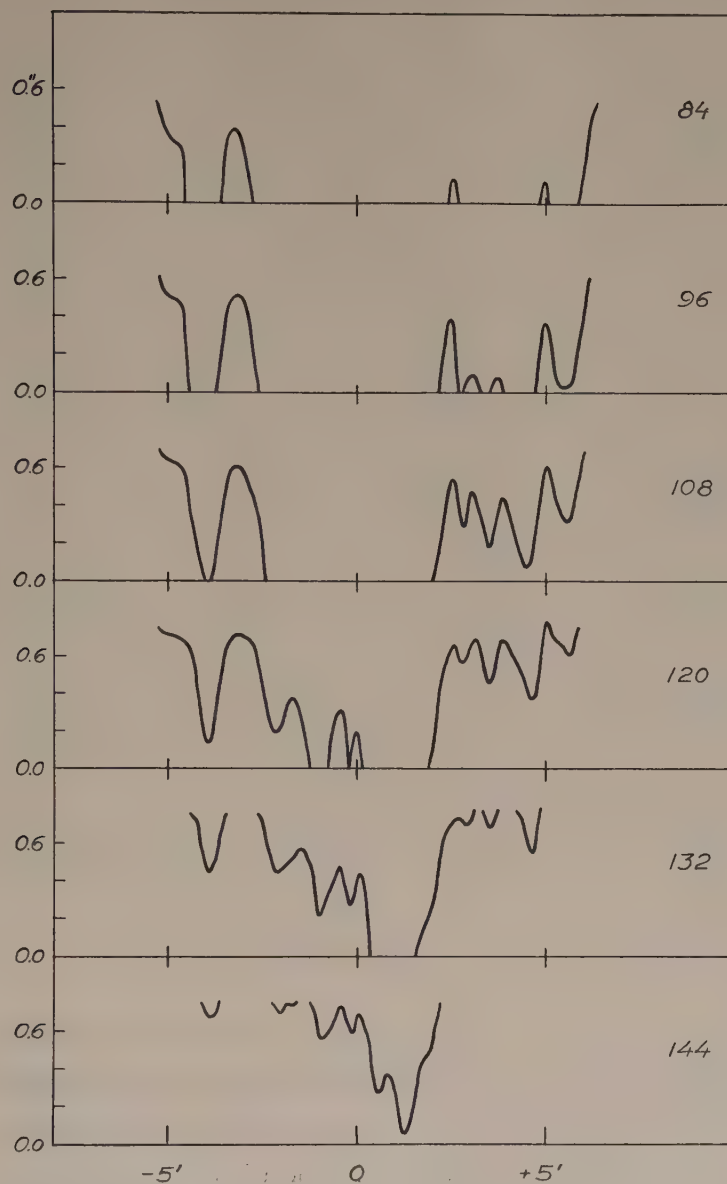


Fig. 23. The position of the photometric contour relative to the photospheric level (horizontal axis) at the moments of some selected exposures. Current numbers are given to the right. Two consecutive exposures differ in time by slightly more than $0^s.5$. The provisional unit of abscissa is $1'$ (lunar apparent radius = $15'.99$). Figure constructed by LINDBLAD on basis of the registration of the second contact.

LINDBLAD used one minute of arc as provisional unit of the abscissae of the photometric tracings (lunar topocentric mean diameter = distance between the coronal stripes = $31'.98$). The photometric heights h_i , in the sense used in (a) above, were then obtained by interpolation from the drawn profiles and for intervals of x equal to $0'.2$ or $0'.72$ in position angle.

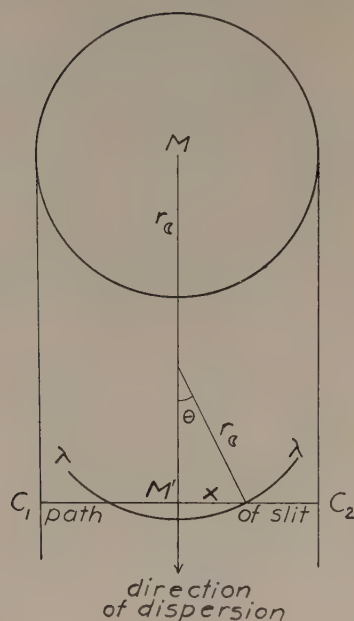


Fig. 24. Conversion of the abscissa x into central angle θ . The coronal stripes are indicated by the straight lines touching the lunar border.

It is seen from Fig. 23 that a certain exposure yields only part of the profile, namely that part which at the moment in question is located within the interval of h where the gradient of intensity has actually been studied. The remaining parts of the contacting lunar profile become photometrically available when they in their turn move through this region of the solar atmosphere. The shape of analogous parts of the marginal zone are closely related as it may be expected. We get an illustrative picture of the photometric method of contact determinations if we consider an arbitrary exposure to be our standard exposure and bring all other profiles to cover the standard profile by accounting for the relative motion. One gets in this way a mean lunar contour which is well defined in time and position relative to the adopted zero level. To determine the moments of contact, it remains only to define, in one way or other, a representative mean lunar level, a problem to which we soon will return.

The conversion of the abscissae x into the central angles θ is readily obtained by means of the simple relation (Fig. 24)

$$x = r_{\odot} \sin \theta$$

where the zero point of x is the midpoint M' of the distance $C_1 C_2 (= 2r_{\odot})$ between the two coronal stripes. The angle $\theta = 0$ is thus attributed to the point at the lunar periphery where this is intersected by the line through the lunar centre which runs parallel with the direction of dispersion. The derivation of the central angles is accordingly easily achieved when the photometric tracings have been extended so as to include the coronal stripes as well. This was, as related, the case for one of the expeditions (Stockholm) whereas the other used instead two small prominences as marks of orientation (p. 42). In the latter case, therefore, the scale reading x_0 (on the auxiliary glass plate) of the midpoint M' had to be determined by means of a measuring machine in the red part of the spectrum; the coronal stripes were quite too faint in the blue colour to serve as fix points at these measurements. The principle used was stating with the comparator the abscissa x' , expressed in mm, of some well defined details (hills, valleys) for which the scale reading x in the blue region was already known from the procedure adopted

Table 11

Determination of $C_1 C_2 = 2r_{\zeta}$ (Fig. 24) from the flash spectrum records

Exp. No.	$2r_{\zeta}$ mm	Contact	Exp. No.	$2r_{\zeta}$ mm	Contact
388	6.002	second	44	5.954	third
400	5.971	"	49	5.998	"
410	6.000	"	79	6.008	"
412	6.020	"	94	5.966	"
415	6.003	"	114	6.005	"
39	5.992	third	124	5.991	"
Mean value: $2r_{\zeta} = 5.992 \pm 0.006$ (mm)					

Table 12

Determination of the scale reading x_0 of the midpoint M' (Fig. 24)

Second contact				Third contact			
Detail No.	x' mm	x	x_0	Detail No.	x' mm	x	x_0
1	+0.614	11.81	28.14	1	+0.277	24.06	31.43
2	+ .184	23.23	28.12	2	— .368	41.06	31.27
3	— .027	28.76	28.04	3	— .924	56.00	31.42
4	— .236	34.30	28.02				
Mean			28.08	Mean			31.37

at the determination of the photometric profile. The magnification of scale from original flash spectrum to photometrical tracing was established by registering, with the Koch-Goos photometer, the lines of a carefully manufactured transparent millimeter scale; the procedure mentioned, which accordingly takes into account a possible shrinking of the registering photographic papers in the drying machine, rendered the magnification to be 39.90 times (value stated by the manufacturing firm to be 40 times). Since the spacing of the lines of the auxiliary glass plate was $1\frac{1}{2}$ mm it follows that 1 mm on the flash spectrum record corresponds to 26.60 units of x . The determination of the central angles θ is consequently obtained from the relation

$$(x - x_0) = r_{\zeta} \cdot 26.60 \cdot \sin \theta$$

where r_{ζ} is expressed in mm. Tables 11 and 12 give the results concerning the determination of r_{ζ} and x_0 .

To determine the orientation of the photometric contours with respect to the lunar centre it merely remains to find the position angle p_0 which corresponds to $\theta = 0$. This would be an easy matter if the direction of dispersion was known with sufficient accuracy, say some few tenths of a degree. Such a precision in the adjustment of the prism's refracting edge is, however, not likely to be achieved. Prominences of known orientation might be used for the purpose, although the small scale may raise some obstacles here. The present way of determining the orientation has therefore been to fit as well as possible the photometric contour into the contour derived on the basis of HAYN's topographic map (Fig. 11, p. 34). The procedure in question is, as we shall see, justified by the strikingly good agreement in shape between the two profiles. Every well defined detail to be recognized in the two curves of the limb furnished the zero point $p_0 = p - \theta$ which was found to scatter rather slightly about the mean value $\overline{p_0}$. The final orientation was then established by fitting into each other those portions of the photometric contours which were common to both stations. A valuable control of the final result was provided by the fact that LINDBLAD had his prism unturned during the cinematographic registration and, accordingly, the condition

$$p_{03} - p_{02} = 180^\circ$$

should be fulfilled (additional subscripts denoting the contacts). Actually, the best agreement with HAYN's contour was found for

$$p_{02} = 93^\circ 0,$$

$$p_{03} = 273^\circ 6,$$

which means that a residual $= 0^\circ 6$ still exists. Whether this is to be attributed to the stated orientations themselves or to some other kind of error is not easy to say; it may, at any rate, not affect appreciably the determination of contacts. — It should be added that the above procedure was capable of realization only after a reduction of the photometric contours to a provisional mean lunar level which facilitated considerably the inter-comparison of the different profiles.

The results of the contour determinations are given in Table 13 and 14 at the end of this chapter. The individual statements of the reduced photometric heights h_{0i} , printed in italics, are given there for contour points of abscissae x , central angles θ and position angles p . The row headed h_0 gives the mean value of the reduced photometric heights, with the corrections Δh already included (see also Explanations to the Tables, p. 83). The corresponding profiles are visualized in Figs. 26 a and 27 a (pp. 70 and 71).

It is seen from the tables that for both expeditions the height values display the relative constancy which we should expect to find. In Table 14, third contact, there is a small trend in the given values, however, which might be explained by the presence of a slight background intensity.

45. *Finding the centre and the mean level of the lunar contours.* The wings of the curves in Figs. 26 a and 27 a bend upwards from the horizontal axis (which represents the photospheric level) on account of the different curvatures of the borders of the Sun and the Moon. This effect will be removed by fitting into the photometric contours a mean lunar level and by referring the contour points to this level instead. An operation like this will at the same time furnish the well defined lunar border which is needed at the determination of the inner contacts. Fig. 25 aims at showing the principle adopted for this purpose.

The dashed curve in this figure represents the shape and position of the Moon's border at the moment t_{02} , established in the way outlined in the foregoing section. Accordingly, for an arbitrary contour point Q are known the height h_0 above the adopted solar edge and the position angle p , as seen from the lunar centre M . It is required to find the difference $(r_{\odot} - r_{\odot})$ in topocentric radius between the two celestial bodies and, moreover, the position of M in terms of the rectangular coordinates (x, y) of origin S (solar centre). With h is denoted the height of Q above the introduced mean level.

By identifying the projections of SQ and SM upon QM with the length of the latter, one gets $\pi = 180^\circ$

$$(r_{\odot} + h_0) \cos \varepsilon + d \cos (p_0 - \pi - p) = r_{\odot} + h$$

where (d, p_0) are the polar coordinates of M with respect to S . Developing $\cos \varepsilon$ to the second power of the argument and neglecting the very small term $h_0 \frac{\varepsilon^2}{2}$ one obtains

$$d \cos (p_0 - p) + (r_{\odot} - r_{\odot}) = h_0 - r_{\odot} \cdot \frac{\varepsilon^2}{2} - h.$$

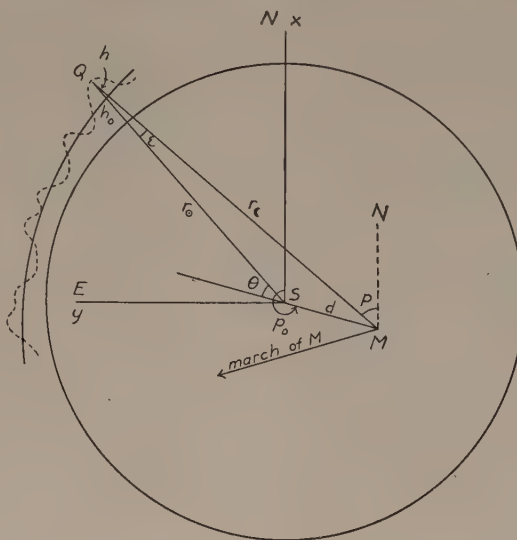


Fig. 25. Determination of centre and mean level of the photometrically established contours.

Putting

$$\begin{aligned} d \cos p_0 &= x_2, & \cos p &= a, & r_{\odot} - r_{\zeta} &= z \\ d \sin p_0 &= y_2, & \sin p &= b, & r_{\odot} \cdot \frac{\varepsilon^2}{2} &= \nu \end{aligned}$$

we get

$$a x_2 + b y_2 + z_2 = h_0 - \nu - h \quad (3)$$

and, similarly, at the third contact

$$a x_3 + b y_3 + z_3 = h_0 - \nu - h. \quad (4)$$

The small correction term ν , which may be written $(d^2/2r_{\odot}) \cdot \sin^2 \theta$, is found with sufficient approximation by putting $d = r_{\odot} - r_{\zeta}$ (near contact) and $r_{\odot} = 16'$. Actually, for $\theta = 30^\circ$ and $r_{\odot} - r_{\zeta} = 15''.5$ (according to the predictions by GRÖNSTRAND [6]), the value of ν amounts to only $0''.03$.

The quantities x , y and z have to be evaluated from a least squares solution applied to the equations of condition (3, 4). The six unknowns are however not independent of each other but related in the following simple way

$$\begin{aligned} x_3 &= x_2 + \xi, \\ y_3 &= y_2 + \eta, \\ z_3 &= z_2 + \zeta, \end{aligned} \quad (5)$$

where ξ , η , ζ are the increments in x , y , z during the involved time interval $(t_{03} - t_{02})$. Making the linear substitution (5) in the equation (4) we get a least squares determination of x_2 , y_2 , z_2 , based upon the contours from both contacts. Hence, the equations of condition may be written

$$\begin{aligned} a x_2 + b y_2 + z_2 &= h_0 - h_H - \nu - h_1 \text{ (second cont.)} \\ a x_2 + b y_2 + z_2 &= h_0 - h_H - \nu - a\xi - b\eta - \zeta - h_1 \text{ (third cont.)} \end{aligned} \quad (6)$$

where, to increase the accuracy of the solution, h_0 has been replaced by $h_0 - h_H$, h_H denoting the height of Q as obtained from the topographic map of HAYN. One renders in this way the level, to be established from (6), in better agreement with the true mean level and, likewise, the coordinates (x_2, y_2) of the centre to be closer to the true centre. The residual height h_1 measures the remaining roughness of the photometric contour since this has been smoothed by applying the heights h_H . The final height of Q relative to the level defined by (6), will accordingly be

$$h = h_H + h_1.$$

The quantities ξ , η in (5) denote the rectangular displacements of the lunar centre during the interval $(t_{03} - t_{02})$. With the notations used on p. 44, we have

$$\xi = [\Delta\alpha(t_{03}) - \Delta\alpha(t_{02})] \cos \delta_{\odot},$$

$$\eta = \Delta\delta(t_{03}) - \Delta\delta(t_{02}),$$

or, if v denotes the numerical value of the relative velocity at maximum eclipse, with sufficient accuracy

$$\xi = v(t_{03} - t_{02}) \cos \phi,$$

$$\eta = v(t_{03} - t_{02}) \sin \phi,$$

where ϕ is the position angle of the relative velocity vector. The quantity ζ denotes the change in lunar topocentric radius caused by the slightly varying topocentric daily parallax of the Moon. The rate of change of the latter, $-0''.028 \text{ min}^{-1}$, renders the increment in question to be

$$\zeta = -0''.028 \cdot (t_{03} - t_{02}) \cdot \frac{1}{60}$$

where the time t should be expressed in seconds.

The least squares solution has been based on all contour points at which the photometric heights h_0 have been derived (in all, some 110 for each expedition). The primary data and the results of the calculations are given in Table 15 (cf. Table 5, p. 44).

The resulting limb irregularities h , now referred to the level of the mean contour, are given at the end of this chapter in Tables 13 and 14, respectively. To show the numerical agreement with the corresponding statements by HAYN the residual heights $h_1 (= h - h_H)$ have also been given there. Figs. 26 b and 27 b show the reduced photometric profiles as superposed on the corresponding profiles of HAYN. Finally, Fig. 28 shows the reduced contours — thus obtained independently from the Lund and Stockholm photometric materials — drawn on the same horizontal axis; the agreement between them may be studied within those parts of the lunar border which they have in common.

46. *Intercomparison of the photometric profiles. Accuracy in the determination of relative contacts.* A glance at Fig. 28 reveals that the lunar contours derived from the materials of the two expeditions, as may be judged from their common parts, agree very well with each other. A numerical estimate of this agreement is found from a study of the residuals ($L = \text{Lund}$, $S = \text{Stockholm}$)

$$\Delta h = h_L - h_S$$

Table 15

Primary data and results of the least squares solution aimed at finding the centre and mean level of the photometric contours

	Lund Exp.	Stockholm Exp.
v	$0''.4701 \text{ sec}^{-1}$	$0''.4703 \text{ sec}^{-1}$
ϕ	$95^\circ 73$	$95^\circ 80$
ξ	$-2''.763$	$-3''.139$
η	$27''.532$	$30''.973$
ζ	$-0''.027$	$-0''.031$
t_{02}	$14^{\text{h}}00^{\text{m}}28^{\text{s}}.20 \text{ U.T.}$	$14^{\text{h}}00^{\text{m}}58^{\text{s}}.79 \text{ U.T.}$
t_{03}	$14^{\text{h}}01^{\text{m}}27^{\text{s}}.06 \text{ U.T.}$	$14^{\text{h}}02^{\text{m}}04^{\text{s}}.99 \text{ U.T.}$
x_2	$-4''.57 \pm ''11$	$+1''.16 \pm ''09$
y_2	$-14''.454 \pm ''022$	$-15''.628 \pm ''021$
z_2	$+16''.067 \pm ''044$	$+16''.210 \pm ''022$

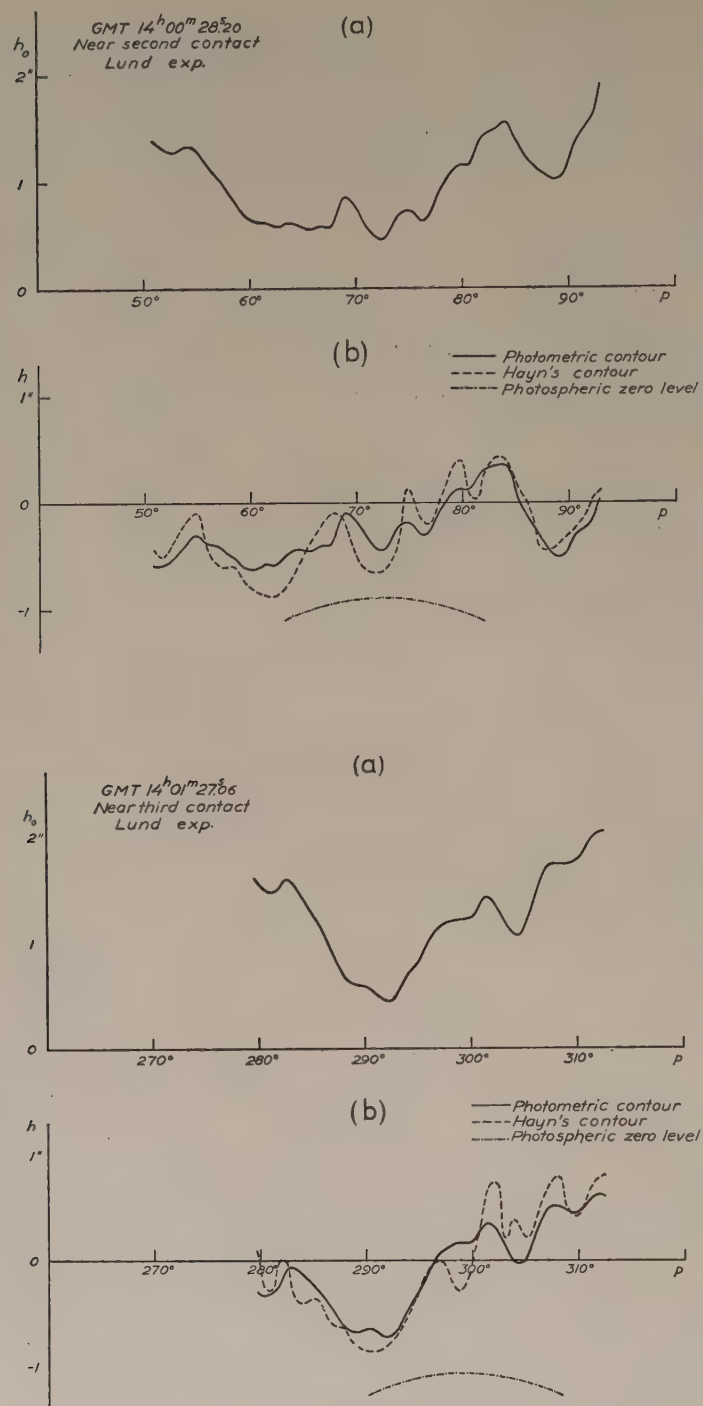


Fig. 26. The photometric contours at second and third contacts (Lund expedition); (a) contour referred to photospheric level (b) contour referred to mean lunar level.

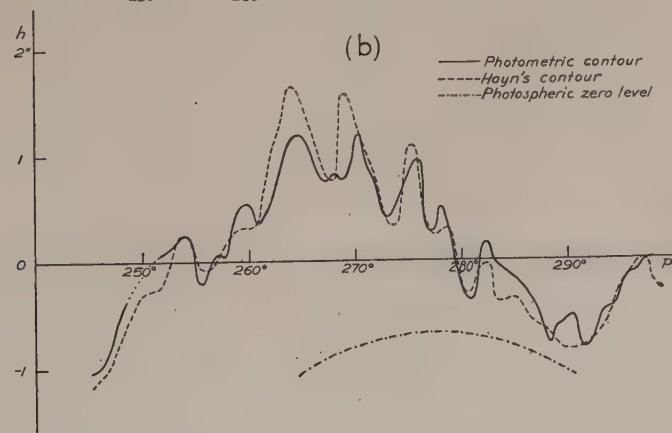
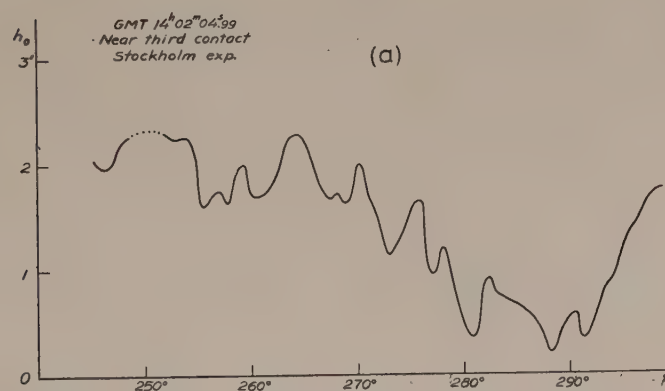
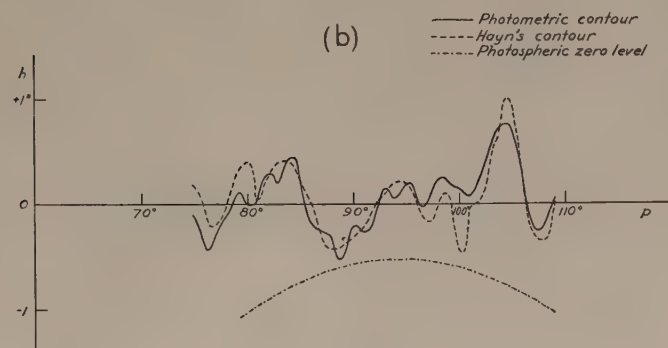
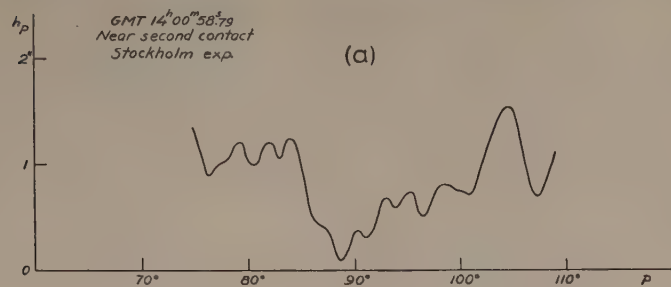


Fig. 27. The photometric contours at second and third contacts (Stockholm expedition); (a) contour referred to photospheric level (b) contour referred to mean lunar level.

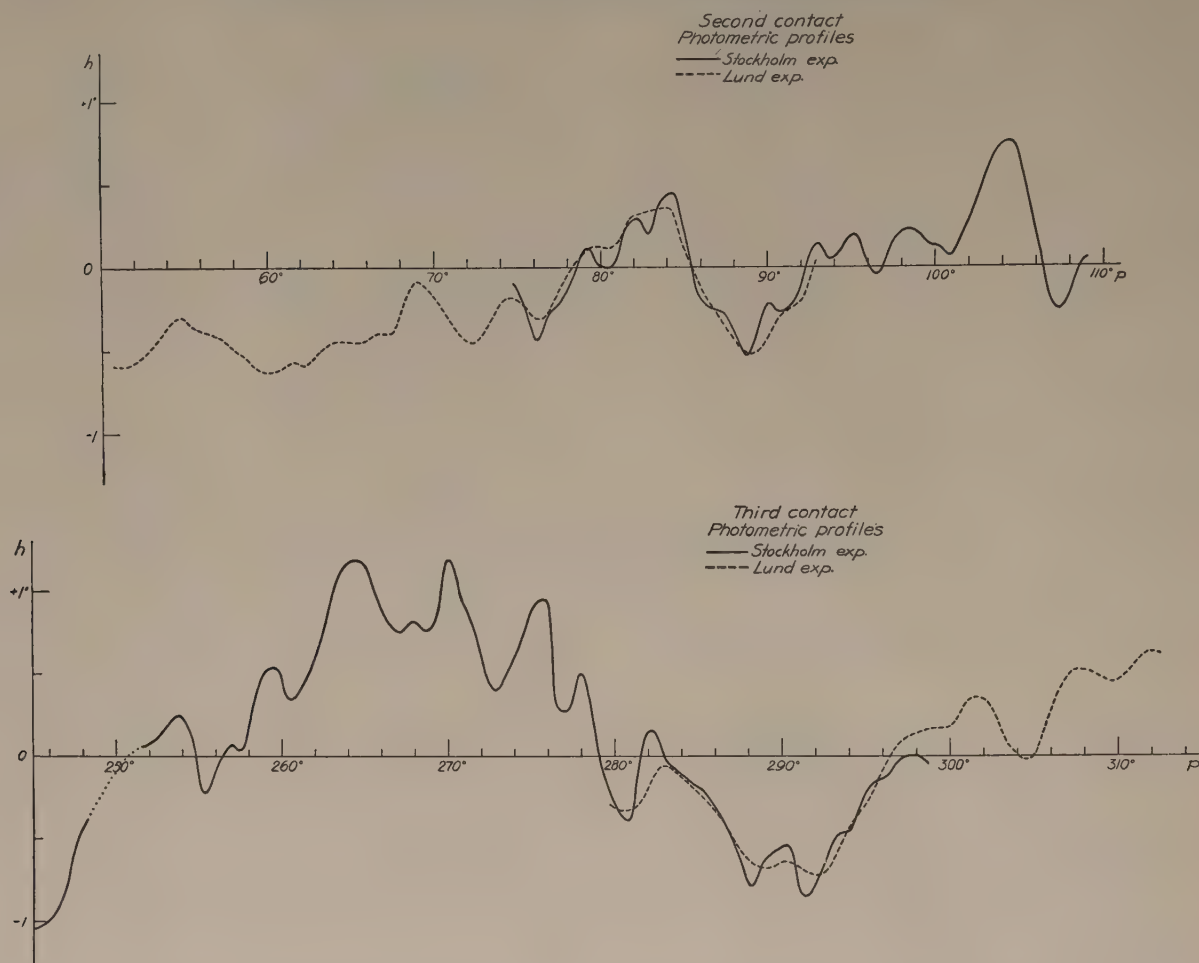


Fig. 28. The photometric contours as derived from the spectral registrations of the Lund and the Stockholm expeditions. The horizontal axis represents mean lunar level.

which have been listed in Table 16 for equidistant values of the position angle p . It is seen from this table that the quantity Δh with the exception of three cases only does not exceed $0''.15$ and generally assumes much smaller values. From its change with increasing position angle we are not able to state conclusively any systematical run of Δh which otherwise would have indicated a relative tilt of the two mean levels. Such an effect might have appeared from the fact that the profiles have been derived independently on the basis of partly differing (and rather limited) portions of the lunar border; the intermediary use of HAYN'S contour which in fact aimed at eliminating the greatest part of the influences from the actual topography has, of course, contributed essentially towards agreement between the two solutions. On the other hand, Fig. 28 reveals that the extreme wings of the contours display a small but yet discernible depression. This is a natural consequence of the fact that these parts of the lunar border can be

Table 16

Second contact				Third contact			
p	h_L	h_S	$h_L - h_S$	p	h_L	h_S	$h_L - h_S$
74.7	-.20	-.09	-.11	279.7	-.31	-.20	-.11
75.4	-.23	-.22	-.01	280.4	-.34	-.36	+.02
76.1	-.31	-.43	+.12	281.2	-.31	-.28	-.03
76.9	-.27	-.30	+.03	282.0	-.20	+.14	-.34
77.6	-.12	-.20	+.08	282.7	-.08	+.07	-.15
78.3	.00	-.08	+.08	283.5	-.09	-.07	-.02
79.0	+.08	+.08	.00	284.2	-.15	-.14	-.01
79.7	+.12	+.07	+.05	284.9	-.21	-.19	-.02
80.5	+.09	.00	+.09	285.7	-.29	-.27	-.02
81.2	+.17	+.14	+.03	286.4	-.40	-.38	-.02
81.9	+.31	+.28	+.03	287.2	-.52	-.53	+.01
82.6	+.32	+.25	+.07	287.9	-.63	-.76	+.13
83.3	+.32	+.32	.00	288.6	-.67	-.69	+.02
84.1	+.33	+.43	-.10	289.4	-.66	-.59	-.07
84.8	+.16	+.34	-.18	290.1	-.64	-.54	-.10
85.5	-.01	-.01	.00	290.8	-.66	-.64	-.02
86.2	-.15	-.21	+.06	291.5	-.70	-.83	+.13
87.0	-.27	-.26	-.01	292.3	-.71	-.72	+.01
87.7	-.38	-.35	-.03	293.0	-.65	-.52	-.13
88.4	-.48	-.52	+.04	293.7	-.50	-.47	-.03
89.2	-.52	-.47	-.05	294.4	-.37	-.36	-.01
89.9	-.46	-.23	-.23	295.2	-.26	-.20	-.06
90.7	-.30	-.26	-.04	295.9	-.10	-.14	+.04
91.4	-.25	-.24	-.01	296.6	+.02	-.08	+.10
92.1	-.21	-.09	-.12	297.3	+.10	-.01	+.11
92.9	+.03	+.13	-.10	298.0	+.14	-.01	+.15

studied photometrically only when the more central parts of the continuous flash spectrum are rather strongly overexposed and, hence, light scattered in the instrument or in the emulsion itself will add to the ordinary intensities at the borders of the photospheric spectrum.

However, a more conspicuous and at the same time more important point is that the different features of the L -contour turn out to be rather more level than the corresponding features of the S -contour. This divergency is to be attributed to the different scales of the recording instruments, the L -scale being in fact only some two-thirds of the other ($f_L = 64$ cm, $f_S = 101$ cm). It is reasonable to assume that by using equal or nearly equal focal lengths of the light gathering instruments we would have attained

an even better agreement between the photometric contours. The relative or geodetic contact problem, which makes use of the difference in contact times which are found at the ends of a long geodetic basis, will thereby be solved with an even more satisfactory exactitude as we are now going to see.

The least squares determination of the coordinates x_2, y_2 yielded the position of the lunar centre relative to the solar centre at the known moment t_{02} (Table 15). That the mean error at this computation turned out to be considerably greater for the x -coordinate (about 0".10) than for the y -coordinate (about 0".02) is not surprising in view of the principle underlying the determination of the relative position of the solar and lunar centres. For the establishment of the contact times, since the relative motion follows nearly the direction west-east, the accuracy of the stated y -coordinate is naturally of greater importance. Obviously, however, the actual mean error 0".02 does not result from errors in the photometric measurements of the lunar contours in proper sense but from the divergency between the photometric heights and the heights according to H_{AYN}. The least squares determination of the coordinates of the lunar centre constitutes what may be called an attempt to an absolute solution of the contact problem where the formal accuracy will be settled by the deviation of the lunar edge — smoothed by applying H_{AYN}'s corrections or not — from circular shape. In the present relative contact problem, on the other hand, we are concerned only with the question to what degree the two lunar centres as merely defined by the actual photometric contours really agree with each other. When considered in this way, the present problem reduces to a question of the mutual agreement of the photometric contours of the two expeditions.

The primary data needed for an investigation of this kind is already contained in Table 16. Table 17 gives, on the basis of this material, the following data obtained in considering Δh as a quantity with Gaussian distribution:

$$\begin{aligned} H &= \overline{\Delta h} = \text{mean value of the differences } (h_L - h_S), \\ \varepsilon(\Delta h) &= \text{m.e. in one determination of } (h_L - h_S), \\ \varepsilon(H) &= \text{m.e. in the mean of } (h_L - h_S), \\ \varepsilon(h) &= \varepsilon(\Delta h) : \sqrt{2} = \text{m.e. (external) in one photometric determination of height} \\ &\quad \text{derived on the basis of equal weights of } h_L \text{ and } h_S. \end{aligned}$$

Two cases have been considered bearing upon the utilized portions of the lunar contours; in (a) the entire common contours have been employed whereas (b) refers to the case that, in agreement with an above statement, the extreme parts of the common borders have been excluded.

Table 17

Contact	Interval of p	H	$\varepsilon(\Delta h)$	$\varepsilon(H)$	$\varepsilon(h)$
Second	(a) 74°7—92°9	—0"012	0"086	0"017	0"061
	(b) 76.9—90.7	— .004	.085	.019	.060
Third	(a) 279.7—298.0	— .016	.103	.020	.073
	(b) 282.7—295.9	— .017	.072	.016	.051

With regard to the affecting mean errors, the absolute value $|H|$ of the reduction to common contacting mean level is found to be negligible in all of the four cases. Disregarding this and dealing instead with the general case, the numerical value of the reduction from one centre to the other along the path of the relative motion will be (subscripts referring to the contacts)

$$\Delta l = \frac{H_2 - H_3}{2}$$

found by rendering the mean contours of the two expeditions concentric with each other. The uncertainty to which this reduction is liable is

$$\varepsilon(\Delta l) = \frac{1}{2} \sqrt{\varepsilon^2(H_2) + \varepsilon^2(H_3)},$$

which thus signifies the accuracy in the determination of the common centre as derived on the basis of the mutual agreement between the photometric contours. The time interval during which the axis of the shadow cone sweeps from one observer to the other may in consequence of this be determined with the accuracy (v = relative angular velocity)

$$\begin{aligned} \varepsilon(\Delta t) &= \varepsilon\left(\frac{\Delta l}{v}\right), \\ &= \frac{1}{2v} \sqrt{\varepsilon^2(H_2) + \varepsilon^2(H_3)}. \end{aligned}$$

This expression thus gives the precision in the determination of the time difference.

The reduction Δl , which necessarily must be very small, may be regarded to act in the direction of the y -axis only. Accordingly, if we consider x_{2L} , y_{2L} as representing the position of our standard centre, the coordinates of this centre in the second case will be

$$x_{2S}, y_{2S} + \Delta l.$$

Actually, as found from Table 17, the correction Δl amounts to but some few thousandths of a second of arc.

Table 18

	$\varepsilon(\Delta l)$	$\varepsilon(\Delta t)$
(a)	0".0131	0 ^s .028
(b)	.0125	.027

The quantities $\varepsilon(\Delta l)$ and $\varepsilon(\Delta t)$ found from the above expressions are given in Table 18 where v has been taken equal to 0".470 sec⁻¹ and the rows headed (a) and (b) refer to the contours dealt with in the foregoing table. Remaining data is likewise to be found in this table.

According to BONSDORFF, a geodetic application of the eclipse method might prove useful for further inferences provided the time difference could be stated with an accuracy of 0^s.028, not considering the error arising from the interfering lunar topography (p. 6). The latter was estimated by BONSDORFF to be about 30 m in the distance on the Earth or, approximately, 0^s.03 in the time difference. The total error would thereby be allowed to attain a value of about 0^s.035. From a comparison with the corresponding precision measures given in Table 18 we find the accuracy attained at the present attempt to fulfil quite satisfactorily this condition in both of the considered cases.

It should be pointed out immediately that the above results have been derived under conditions which have been far from ideal with respect to two particularly important points. Firstly, only a limited portion of the lunar contour has been available for an intercomparison of the photometric profiles due to the fact that one of the expeditions was located some distance from the central line of the total belt; it is clear, moreover, that the concerned portion which embraces the wings of the photometric contours cannot be as suitable for this particular purpose as the more central parts of the separate profiles. Secondly, as has already been emphasized above, the use of different scales has to a considerable extent increased the error $\varepsilon(H)$ and thereby diminished the final precision. We may conclude therefore that with such obstacles removed the accuracy will be much greater than actually found. It seems in fact probable that under more favourable circumstances the error would be kept as low as 0^s.015.

It goes without saying that the figures in Table 18 will yield a correct measure of the accuracy only when no systematic errors of influential size interfere. Such errors can arise from a possible change in the position of the photospheric level which has been made use of for the purpose of defining a sharp solar edge. We have seen before however that with some simple precautions taken errors of this kind may be kept under effective control and if necessary accounted for (p. 57). Of course, misstatements in the timing of the spectral records at any one or both of the contacts will likewise imply an introduction of systematic errors.

47. *An attempt to check the results obtained at the two stations of observation. The empirical corrections to the lunar tabular position.* The least squares determination of the Moon's centre and mean level yielded at the same time the position of the latter relative to the photospheric level at the two specified moments t_{02} and t_{08} . The results are illustrated in Figs. 26 b and 27 b in which the lower (dashed) circular contours indicate the momentary positions of the photospheric border. The moments of contact, when desired, may readily be determined from the distance between the concerned levels and their relative motion.

Now, actually, the geodetic basis this time was very short, about 30 km, forming moreover with the direction of the central line an angle of very nearly 30° . It is clear that such a basis does not allow the inference of any geodetic data in proper sense, nor does it offer any particularly favourable conditions for a determination of the basis from the results obtained at the two stations whereby, otherwise, a valuable check of the relative contact times would have been realized. What we primarily can attain from the determinations of contact is namely not the length of the basis itself but the distance between those points on the central line where this is cut by two isochrones of (say) maximum phase, passing through the points of observation. A complete solution of the geodetic problem requires, besides, an accurate knowledge of the real path of the central line which, on the other hand, presumes additional observations to be carried out close to the northern and southern borders of the total belt. We shall return briefly to this question in next section and confine ourselves now to stating that we here lack more precise data concerning the path of the central line. In consequence of this and of the fact that for one of the expeditions the registered times may have been somewhat erroneous we shall be content with comparing the values of the empirical corrections to the tabular positions as found from the two separate solutions.

The expressions on p. 44 give the tabular distances between the lunar and solar centres which can now be directly compared to those stated from the actual observations. Since the location of the observation sites are given in terms of geodetically determined coordinates we may at this comparison disregard possible local deflections from the vertical. The connection between the topocentric differences in equatorial coordinates and the rectangular coordinates (x_2, y_2) of the lunar centre is

$$\begin{aligned}\delta'_\zeta - \delta'_\odot &= x_2, \\ \alpha'_\zeta - \alpha'_\odot &= y_2 / \cos \delta_\odot,\end{aligned}$$

where (Table 15, p. 69)

$$x_{2L} = -4''.57 \pm 0''.11, \quad x_{2S} = +1''.16 \pm 0''.09,$$

$$y_{2L} = -14.454 \pm 0.022, \quad y_{2S} = -15.628 \pm 0.021,$$

and $\delta_{\odot}$, the apparent declination of the solar centre, is $22^{\circ}22'09$. The calculated and observed differences are given in Table 19 together with the resulting empirical corrections.

Table 19
Determination of empirical corrections to the tabular positions

	t_{02}	$\alpha'_{\zeta} - \alpha'_{\odot}$		$\delta'_{\zeta} - \delta'_{\odot}$		Emp. corr. (obs.-calc.)	
		obs.	calc.	obs.	calc.	$\Delta(\alpha_{\zeta} - \alpha_{\odot})$	$\Delta(\delta_{\zeta} - \delta_{\odot})$
Lund exp.	+0 ^m .4701	-15''.630	-14''.303	-4''.57	-3''.58	-1''.327	-0''.99
Stockholm exp...	0.9798	-16.900	-15.638	+1.16	+1.69	-1.262	-0.53

The corrections in declination are seen to differ by 0''.46 which is somewhat more than would be expected from the individual mean errors given above. This is not astonishing, however, in view of the fact that the setting of the centres had to be based on partly different portions of the lunar marginal zone whereby possible systematic errors in HAYN's contour will, above all, be reflected in the difference in declination. For several reasons (central line position of the S-expedition, better agreement with HAYN's contour), the S-value should be regarded as the most trustworthy whence we shall accept

$$\Delta(\delta_{\zeta} - \delta_{\odot}) = -0''.53 \pm 0''.09$$

as the final value of the empirical correction in declination.

As to the corrections in right ascension, these are found to agree to the first decimal place, so that we at any rate can state the correction to be $-1''.3$. Since, for reasons mentioned above, the L-value has to be given a considerably less weight we shall accept also here the S-value and accordingly put

$$\Delta(\alpha_{\zeta} - \alpha_{\odot}) = -1''.26 \pm 0''.023.$$

If the discrepancy $-0''.065$ (L—S) is attributed solely to an error ΔT in the times registered at the Lund expedition we should have very nearly (relative motion forming a small angle with the y -axis)

$$\Delta T = - \frac{2 \cdot 0.065 \cdot \cos \delta_{\odot}}{v}$$

or

$$\Delta T = -0^s.26 \pm 0^s.05$$

an amount to be added to the times actually registered at the third contact. This result does not contradict with a previous estimate made on p. 18.

The empirical corrections adopted in *The Nautical Almanac* (1945) corresponded to $+1''$ in the mean longitude of the Sun as found from Newcomb's *Tables of the Sun* and to $-1''$ in the mean longitude and $-0''.5$ in the latitude of the Moon as found from Brown's *Tables of the Moon*. The figures below give the corresponding relative corrections converted into equatorial coordinates and, besides, the values found from the present investigation.

	Naut. Alm.	Present value
$\Delta(\alpha_{\zeta} - \alpha_{\odot})$	$-2''.24$	$-1''.26 \pm 0''.023$
$\Delta(\delta_{\zeta} - \delta_{\odot})$	-0.36	-0.53 ± 0.09

From a comparison of the corrections in right ascension we may conclude that the eclipse has occurred slightly more than 2^s earlier than would have been expected from the predictional data given in *The Nautical Almanac*. The agreement in declination, finally, is satisfactory with regard to the mean error affecting the observed quantity.

48. *A review of the results obtained in the foregoing sections. On the geodetic project in proper sense.* It has been shown in section 46 that the time needed by the shadow cone to move from one observer to the other could (formally) be determined with an accuracy of $0^s.027$ (Table 18). In the present case, with the velocity of the shadow amounting to 920 m sec^{-1} , this corresponds to an accuracy of 25 m on the surface of the Earth. It has been pointed out in the same section that under more favourable circumstances the accuracy would probably have been something like $0^s.015$, or some 15 m in the distance. It is questionable if the efficiency of the spectrophotometric procedure thus displayed might be rivalled by that of any other procedure, proposed for the same purpose (cf. Chapter I).

The proceeding used here has been to determine, for any one of the observers, the quantities $\Delta(\alpha_{\zeta} - \alpha_{\odot})$ and $\Delta(\delta_{\zeta} - \delta_{\odot})$ in the sense observed—calculated (Table 19). The geodetic basis this time was known very accurately from the geodetic coordinates of its endpoints and, consequently, the local deviations from the vertical should not come into account with appreciable size. If it had not been for a defective registration of time at one of the stations of observation, the quantities mentioned should accordingly have been expected to agree for the two observers. And this would be but another way of expressing the fact that the geodetically determined basis agrees with that established by means of the eclipse method.

In the general case when the stations are spread out all along the track of the total belt the same mode of procedure may be used. We thus obtain, for each station,

the two quantities $\mathcal{A}$ and from these are able to form a number of differences (subscripts referring to the results of two different observers)

$$\mathcal{A}\mathcal{A}(\alpha_{\zeta} - \alpha_{\odot}) = \mathcal{A}_1(\alpha_{\zeta} - \alpha_{\odot}) - \mathcal{A}_2(\alpha_{\zeta} - \alpha_{\odot})$$

and similar expressions for the corrections in declination. The residuals $\mathcal{A}\mathcal{A}$, obviously, have to be interpreted in terms of corrections to the geodetic constants adopted at the derivation of the quantities $\mathcal{A}$. This conclusion, of course, holds true only when "extra-terrestrial" sources of error do not enter with influential size into the residuals $\mathcal{A}\mathcal{A}$. Such errors may arise from an imperfect knowledge of the motions of the Earth and the Moon and, likewise, from an erroneous value of the lunar parallax. It seems possible to keep both of these below a certain critical limit at which they will no longer endanger the geodetic inferences to be drawn. The geocentric relative position between Sun and Moon may be calculated with great accuracy, as did SUNDMAN for the present eclipse (p. 11). The lunar parallax, on the other hand, may be improved by means of occultations. A procedure tried by J. A. O'KEEFE [24^a] seems to be very promising in this respect. This consists in having a number of observers timing the occultations photoelectrically along the line, covered by the shadow of a selected feature at the lunar limb.

From the empirical corrections $\mathcal{A}(\alpha_{\zeta} - \alpha_{\odot})$ and $\mathcal{A}(\delta_{\zeta} - \delta_{\odot})$ (thus assumed to be free from the errors mentioned above), the moment T_0 of central eclipse and the distance D (say, along the isochrone T_0) to the central line may be determined by means of the general prediction data. Since the relative motion forms a moderate angle with the direction of increasing right ascension, T_0 will be only slightly dependent on the correction in declination which, on the other hand, is known with a fairly low accuracy. To increase the weights of the quantities $\mathcal{A}$, observations of other kind might be carried out with a view of determining more exactly the distance D . For this purpose, BONSDORFF [4] recommends using a long focal photography of the solar crescents from points situated outside but close to the borders of the total belt. Another procedure would be to undertake a spectral cinematography from stations placed just inside the northern and southern limits (LINDBLAD). In both cases, the different instruments have to be calibrated against each other which may be achieved in the laboratory before and after the eclipse. The latter procedure should be based on an accurate knowledge of the orientation of the photometric profiles, obtained from a comparison with (for instance) HAYN's profiles. In so doing, the setting of the lunar centre would be considerably increased by using improved contours derived according to the method of WATTS and ADAMS [33]. The latter

procedure seems, in fact, almost indispensable also for an estimate of the effect of differential libration on the contact times at any future large scale application of the eclipse method.

49. *The difference in apparent radius between Sun and Moon.* Table 15 gives the following values of $z = r'_{\zeta} - r'_{\odot}$ as found from the two cinematographic recordings

$$z_L = + 16''.067 \pm 0''.044,$$

$$z_S = + 16''.210 \pm 0''.022.$$

A complete agreement (within the limit of the mean errors) between the two figures should not be expected since the photospheric levels $h = 0$ may be somewhat different, though of course only slightly so. Moreover, from reasons mentioned repeatedly in the foregoing, the z_L -value may be slightly erroneous.

The precision of the z_S -value is $0''.022$ (m.e.) which is far better than the accuracy with which the absolute values of the radii are known. This accuracy may be put equal to $0''.10$ in both cases so that the present value of the difference cannot be employed for the purpose of improving any one of the semi-diameters with the aid of the other. However, a certain test may be applied on the basis of the fact that the level $h = 0$ is safely not situated inside the inversion point, the locus of which according to the contrast theory of E. MACH [17] should represent the optical limb for the human eye. Amongst visual determinations of the solar radius, that of A. AUWERS [1] is still regarded as a standard value. This is

$$r_{\odot} = 959''.63 \pm 0''.10$$

at unit distance of the Sun. A discussion by F. HAYN [12] based on measurements of his own and those of several other investigators yielded for the lunar radius the value

$$r'_{\zeta} = 932''.70 \pm 0''.10$$

bearing upon the parallax $57' 2''.70$ at mean distance (BROWN). Though the different statements are more consistent than is indicated by the given mean error, the real accuracy would according to HAYN hardly exceed the stated figure. The topocentric difference in apparent radius, deduced on the basis of these values, is readily found to be

$$z_{\text{calc}} = + 16''.37 \pm 0''.14$$

as calculated for the moment t_{02} at the Stockholm expedition. From a comparison with the z_S -value given above one finds the photospheric zero level to be situated $0''.16$ outside the inversion point. This figure is reasonable even though its numerical value is rather uncertain in view of the comparatively large mean error.

50. *Precision of different procedures aiming at determining the shape of the lunar contour.*

(a) *Spectrophotometric procedure.* A single determination of height by means of the empirically established gradient of intensity is liable to the mean error (p. 62)

$$\varepsilon = 0''.02$$

and, hence, for a moderate number of independently stated heights the mean error in the mean

$$\varepsilon < 0''.01.$$

These figures bear upon the internal errors. The external mean error, found from an intercomparison of two quite independently derived photometric contours, are of greater importance for the relative contact problem. Actually, this error was found to be (Table 17, p. 75)

$$\varepsilon = 0''.06$$

which accordingly is considerably greater. The reason is, essentially, that the concerned contours have been obtained from flash spectrum recordings differing widely as regards to scale.

(b) *Direct measurements on large scale photographs.* The external error, in the sense used above, was found to be (p. 36)

$$\varepsilon = 0''.24.$$

A considerable part of this error was found to be caused by accidental refraction in the terrestrial atmosphere.

(c) *HAYN'S topographic map of the lunar marginal zone.* A comparison of HAYN'S contour with the photometric contour derived from LINDBLAD'S flash spectra, yielded the mean error of a single profile point in the former to be

$$\varepsilon = 0''.21.$$

At this comparison, the photometric contour has been assumed to be correct. HAYN himself has estimated the mean error to be about $0''.25$.

(d) *Procedure of WATTS and ADAMS [33].* The principle is following lines of equal density along the border of photographic records of the Moon. The arrangement is servo-operated and constructs directly a large scale record of the profile. The designers state the external mean error to be

$$\varepsilon = 0''.07.$$

Explanation to Tables 13 and 14

The tables give the results from the photometric determination of the lunar contours as derived on the basis of the material of the Lund and the Stockholm expeditions, respectively. The heads of the different rows have the following significances:

x = abscissa of the photometric tracing straight across the flash spectrum. The unit in Table 13 corresponds to 1.5 mm on the photometric tracing, whereas in Table 14 the abscissae are given in minutes of arc. Cf. pp. 63, 65.

θ = angle along the lunar periphery (central angle). Cf. p. 64.

p = position angle from the lunar centre. Cf. p. 66.

h_{0i} = the individual values of the reduced heights in seconds of arc as obtained from successive exposures. The height values (printed in italics) refer to the standard moments t_{02} and t_{03} listed in Table 15, p. 69.

h_0 = mean value of the reduced heights h_{0i} , *with the small corrections Δh (treated in section 42) already included.*

h = photometric height as referred to mean lunar level. Cf. pp. 67—69.

$h_1 = h - h_H$, where h_H is the corresponding value stated by HAYN.

Table 13. Lund
Second

x	6	7	8	9	10	11	12	13	14	15	16	17	18	19
θ	16°1	15.3	14.6	13.9	13.1	12.4	11.6	10.9	10.2	9.4	8.7	8.0	7.3	6.5
p	92°9	92.1	91.4	90.7	89.9	89.2	88.4	87.7	87.0	86.2	85.5	84.8	84.1	83.3
h_{0i}	1"86	1.52	1.46	1.34	1.13	1.03	1.04	1.08	1.07	1.17	1.25	1.43	1.52	1.46
	1"89	1.53	1.45	1.36	1.10	1.06	1.04	1.03	1.11	1.15	1.28	1.43	1.52	1.48
	1"93	1.55	1.47	1.36	1.14	1.03	1.01	1.06	1.12	1.20	1.25	1.41	1.52	1.48
		1.60	1.48	1.36	1.15	1.02	1.01	1.07	1.10	1.19	1.32	1.42	1.56	1.48
		1.60	1.49	1.35	1.16	1.07	1.04	1.04	1.12	1.24	1.33	1.42	1.61	1.51
		1.64	1.50	1.37	1.17	1.03	1.00	1.07	1.12	1.23	1.33	1.42	1.57	1.54
		1.61	1.48	1.37	1.18	1.03	1.00	1.06	1.14	1.23	1.32	1.44	1.54	1.54
				1.39	1.17	1.06	1.02	1.07	1.16	1.20	1.29			1.50
				1.41	1.15	1.04	1.00	1.06	1.13	1.16	1.28			
					1.15	1.03	1.01	1.06	1.10	1.13	1.28			
					1.15	1.02	1.02							
h_0	1"90	1.59	1.49	1.38	1.16	1.04	1.02	1.06	1.12	1.19	1.29	1.42	1.55	1.50
h	+ "03	— .21	— .25	— .30	— .46	— .52	— .48	— .38	— .27	— .15	— .01	+ .16	+ .33	+ .32
h_1	— "06	— .23	— .11	— .07	— .14	— .14	— .04	+ .07	+ .09	— .11	— .10	— .06	— .07	— .10

x	35	36	37	38	39	40	41	42	43	44	45	46	47	48
θ	—5°0	—5.7	—6.4	—7.1	—7.9	—8.6	—9.3	—10.1	—10.8	—11.5	—12.3	—13.0	—13.7	—14.5
p	71°8	71.1	70.4	69.7	68.9	68.2	67.5	66.7	66.0	65.3	64.5	63.8	63.1	62.3
h_{0i}	"46	.52	.64	.78	.85	.68	.54	.55	.54	.55	.56	.62	.56	.57
	"49	.54	.64	.77	.87	.68	.61	.58	.58	.57	.60	.59	.60	.57
	"50	.54	.66	.80	.86	.70	.57	.58	.56	.56	.60	.62	.58	.59
	"48	.54	.70	.85	.88	.74	.58	.60	.56	.55	.56	.63	.61	.57
	"48	.54	.70	.82	.83	.74	.56	.60	.57	.58	.61	.64	.61	.59
	"49	.55	.75	.81	.85	.73	.58	.56	.59	.59	.61	.65	.64	.60
	"48	.56	.70	.75	.85	.76	.55	.61	.57	.56	.58	.66	.60	.59
	"48	.56	.69	.83	.83	.74	.59	.63	.58	.59	.63	.63	.63	.55
	"51	.59	.62	.78	.86	.69	.62	.58	.60	.59	.65	.63	.60	.63
	"50	.61	.73	.76	.83	.75	.56	.58	.56	.57	.59	.58	.61	.59
h_0		.57	.65		.87	.75	.57	.55	.54	.54	.59		.57	.55
		.57				.73	.56							.52
h_0	"49	.56	.68	.80	.85	.72	.57	.58	.57	.57	.60	.62	.61	.59
h	— "42	— .35	— .24	— .13	— .08	— .23	— .39	— .40	— .43	— .46	— .46	— .47	— .51	— .56
h_1	+ "24	+ .30	+ .35	+ .30	+ .12	— .11	— .27	— .19	— .11	— .04	+ .12	+ .24	+ .30	+ .32

expedition.

contact.

20	21	22	23	24	25	26	27	28	29	30	31	32	33	34
5.8	5.1	4.4	3.7	2.9	2.2	1.5	.8	.1	— .7	— 1.4	— 2.1	— 2.8	— 3.5	— 4.3
82.6	81.9	81.2	80.5	79.7	79.0	78.3	77.6	76.9	76.1	75.4	74.7	74.0	73.3	72.5
1.43	1.44	1.15	1.11	1.11	1.06	1.02	.87	.64	.64	.67	.68	.66	.60	.45
1.46	1.39	1.23	1.15	1.15	1.06	1.02	.88	.67	.62	.67	.69	.65	.53	.48
1.46	1.42	1.23	1.12	1.13	1.09	1.01	.85	.67	.60	.68	.71	.70	.56	.48
1.46	1.41	1.25	1.14	1.15	1.11	.96	.90	.66	.62	.68	.73	.71	.56	.46
1.48	1.42	1.28	1.15	1.16	1.11	.98	.88	.67	.61	.74	.76	.70	.56	.46
1.50	1.44	1.27	1.20	1.19	1.09	1.02	.80	.74	.67	.70	.74	.70	.60	.44
1.46	1.41	1.26	1.18	1.19	1.12	1.00	.82	.72	.63	.74	.76	.69	.59	.44
	1.44	1.27	1.14	1.13	1.13	.96	.85	.68	.63	.75	.76	.67	.55	.44
		1.25	1.15	1.14	1.05	1.00	.83	.73	.66	.66	.70	.69	.57	.48
		1.25	1.15	1.15	1.07	.93	.83	.66	.62	.68	.71	.65	.58	.46
		1.29	1.14	1.13	1.07					.69	.73		.58	
1.46	1.42	1.25	1.15	1.15	1.09	.99	.85	.68	.63	.70	.72	.68	.57	.46
+ .32	+ .31	+ .17	+ .09	+ .12	+ .08	.00	— .12	— .27	— .31	— .23	— .20	— .23	— .34	— .45
— .04	+ .09	+ .15	— .05	— .27	— .22	— .10	— .04	— .07	— .14	— .25	— .32	+ .11	+ .19	+ .19

49	50	51	52	53	54	55	56	57	58	59	60	61	62	63
— 15.2	— 16.0	— 16.7	— 17.5	— 18.2	— 19.0	— 19.7	— 20.5	— 21.3	— 22.0	— 22.8	— 23.6	— 24.4	— 25.2	— 26.0
61.6	60.8	60.1	59.3	58.6	57.8	57.1	56.3	55.5	54.8	54.0	53.2	52.4	51.6	50.8
.62	.62	.64	.66	.75	.91	1.02	1.06	1.24	1.30	1.28	1.25	1.25	1.29	1.35
.61	.61	.64	.72	.80	.90	1.00	1.10	1.22	1.30	1.29	1.26	1.25	1.30	1.39
.62	.63	.66	.68	.83	.89	.99	1.13	1.19	1.31	1.33	1.31	1.29	1.33	1.40
.65	.63	.66	.73	.80	.93	1.02	1.13	1.23	1.32	1.36	1.32	1.32	1.34	1.41
.61	.63	.67	.74	.82	.92	1.04	1.16	1.22	1.35	1.37	1.30	1.30	1.31	
.61	.64	.70	.74	.74	.89	1.05	1.14	1.23	1.34	1.36	1.30	1.29	1.33	
.67	.66	.65	.72	.83	.92	1.07	1.12	1.23	1.35	1.34	1.30	1.29		
.65	.62	.66	.64	.81	.93	1.06	1.14	1.20	1.32					
.62	.62	.57	.68	.76	.97	1.02	1.12	1.22						
.56	.55	.59	.71	.81	.96	1.04	1.11							
	.56	.65	.66	.78	.95	1.01	1.11							
	.63		.70	.85	.90	1.00								
				.83	.91									
				.83	.90									
.63	.63	.65	.71	.81	.93	1.04	1.13	1.23	1.33	1.35	1.31	1.30	1.34	1.41
— .56	— .60	— .62	— .60	— .55	— .48	— .42	— .39	— .36	— .31	— .36	— .46	— .55	— .58	— .59
+ .32	+ .25	+ .19	+ .14	+ .07	+ .12	+ .18	+ .13	— .06	— .21	— .19	— .18	— .12	— .07	— .15

Table 13 (cont.).

Third

x	13	14	15	16	17	18	19	20	21	22	23	24
θ	13°3	12.6	11.9	11.1	10.4	9.6	8.9	8.2	7.5	6.7	6.0	5.3
p	312°5	311.8	311.1	310.3	309.6	308.8	308.1	307.4	306.7	305.9	305.2	304.5
h_{0i}	2"02	1.99	1.87	1.74	1.70	1.73	1.73	1.71	1.57	1.27	1.09	1.04
	2"06	2.03	1.89	1.79	1.73	1.70	1.70	1.68	1.62	1.31	1.11	1.10
			1.92	1.82	1.76	1.73	1.74	1.74	1.63	1.39	1.12	1.07
			1.92	1.84	1.76	1.75	1.74	1.73	1.54	1.38	1.17	1.07
			1.94	1.80	1.73	1.75	1.76	1.72	1.62	1.36	1.10	1.11
				1.82	1.74	1.72			1.62	1.39	1.16	1.08
						1.74			1.61	1.38	1.21	1.06
										1.41	1.16	1.09
										1.40	1.15	1.05
										1.33	1.16	1.06
											1.15	1.05
												1.09
												1.09
h_0	2"05	2.01	1.91	1.80	1.74	1.73	1.73	1.72	1.60	1.36	1.14	1.07
h	+ "60	+ .60	+ .54	+ .47	+ .46	+ .48	+ .51	+ .53	+ .43	+ .22	+ .02	— .03
h_1	— "20	— .17	— .11	+ .03	+ .04	— .08	— .28	— .19	— .15	— .13	— .19	— .34

x	38	39	40	41	42	43	44	45	46	47	48	49
θ	— 4°8	— 5.5	— 6.2	— 6.9	— 7.7	— 8.4	— 9.1	— 9.8	— 10.6	— 11.3	— 12.0	— 12.8
p	294°4	293.7	293.0	292.3	291.5	290.8	290.1	289.4	288.6	287.9	287.2	286.4
h_{0i}	"69	.54	.46	.47	.50	.50	.55	.57	.57	.69	.81	1.02
	"72	.58	.50	.43	.45	.56	.60	.61	.64	.67	.83	1.01
	"72	.62	.52	.47	.51	.54	.58	.60	.62	.74	.82	1.03
	"73	.65	.53	.47	.50	.56	.59	.61	.63	.72	.87	1.01
	"75	.64	.48	.45	.48	.53	.59	.62	.66	.72	.88	1.07
	"75	.60	.50	.44	.46	.62	.65	.65	.67	.71	.85	1.05
	"70	.60	.42	.45	.51	.53	.59	.64	.66	.75	.87	1.04
	"80	.64	.44	.38	.44	.54	.60	.61	.62	.70	.85	.99
	"68	.62	.47	.42	.46	.54	.57	.57	.64	.72	.90	1.03
	"71	.57		.42	.44	.56	.61	.60	.59	.71	.86	1.05
	.66					.56	.57	.56	.62	.69	.88	
							.61	.60				
h_0	"73	.61	.48	.44	.48	.55	.59	.60	.63	.71	.86	1.03
h	— "37	— .50	— .65	— .71	— .70	— .66	— .64	— .66	— .67	— .63	— .52	— .40
h_1	+ "06	+ .06	+ .05	+ .06	+ .14	+ .20	+ .21	+ .15	+ .10	.00	+ .10	+ .16

Lund expedition.

contact.

25	26	27	28	29	30	31	32	33	34	35	36	37
4.6	3.9	3.1	2.4	1.7	1.0	.3	— .4	— 1.2	— 1.9	— 2.6	— 3.3	— 4.0
303.8	303.1	302.3	301.6	300.9	300.2	299.5	298.8	298.0	297.3	296.6	295.9	295.2
1.14	1.17	1.26	1.37	1.33	1.21	1.20	1.19	1.15	1.13	1.05	.92	.81
1.11	1.20	1.30	1.38	1.35	1.20	1.20	1.21	1.17	1.16	1.11	.98	.80
1.13	1.17	1.34	1.43	1.37	1.26	1.22	1.19	1.19	1.16	1.06	.95	.82
1.11	1.20	1.27	1.43	1.36	1.24	1.22	1.23	1.20	1.17	1.07	.99	.85
1.13	1.23	1.31	1.43	1.35	1.28	1.24	1.23	1.19	1.16	1.05	.95	.81
1.14	1.18	1.38	1.42	1.37	1.28	1.21	1.23	1.21	1.15	1.13	.98	.88
1.08	1.17	1.37	1.42	1.34	1.24	1.15	1.22	1.20	1.17	1.06	.96	.78
1.08	1.29	1.41	1.39	1.35	1.23	1.19	1.17	1.15	1.13	1.09	1.00	.84
1.16	1.22	1.41		1.35	1.20	1.17	1.20	1.18	1.13	1.09	.94	.80
1.13	1.27	1.41			1.20	1.18	1.19	1.17		1.08	.96	.82
1.16	1.26	1.42			1.24	1.23	1.19				1.00	.82
												.85
1.12	1.21	1.35	1.41	1.35	1.23	1.20	1.20	1.18	1.15	1.08	.97	.82
+ .03	+ .14	+ .29	+ .36	+ .30	+ .19	+ .16	+ .16	+ .14	+ .10	+ .02	— .10	— .26
— .35	— .06	— .45	— .35	— .06	+ .13	+ .36	+ .45	+ .32	+ .11	+ .03	+ .01	— .04
50	51	52	53	54	55	56	57	58	59	60	61	62
— 13.5	— 14.3	— 15.0	— 15.7	— 16.5	— 17.2	— 18.0	— 18.8	— 19.5	— 20.3	— 21.1	— 21.8	— 22.6
285.7	284.9	284.2	283.5	282.7	282.0	281.2	280.4	279.7	278.9	278.1	277.4	276.6
1.14	1.28	1.36	1.45	1.54	1.51	1.44	1.45	1.52	1.60	2.03	2.11	2.26
1.17	1.27	1.38	1.47	1.54	1.50	1.44	1.46	1.51	1.63	2.02	2.16	2.36
1.18	1.26	1.39	1.56	1.58	1.52	1.45	1.48	1.59	1.72	2.06	2.21	2.38
1.16	1.30	1.37	1.51	1.59	1.54	1.49	1.53	1.55	1.77	2.10	2.31	
1.15	1.32	1.39	1.54	1.60	1.53	1.47	1.51	1.60	1.79	2.22	2.33	
1.17	1.28	1.49	1.57	1.64	1.52	1.48	1.52	1.65	1.88	2.21		
1.21	1.28	1.41	1.54	1.63	1.54	1.48	1.53	1.62	1.88			
1.12	1.37	1.42	1.56	1.60		1.49	1.53	1.71	1.87			
1.13	1.28	1.46		1.62			1.57	1.69	1.94			
1.24	1.31	1.43							2.04			
	1.34								2.04			
1.18	1.31	1.42	1.53	1.60	1.53	1.48	1.52	1.61	1.84	2.12	2.23	2.35
— .29	— .21	— 1.5	— .09	— .08	— .20	— .31	— .34	— .31				
+ .12	+ .15	+ .25	+ .31	+ .02	— .20	— .03	— .06	— .41				

Table 14. Stockholm
Second

x	-4.4	-4.2	-4.0	-3.8	-3.6	-3.4	-3.2	-3.0	-2.8	-2.6	-2.4	
θ	-16.0	-15.2	-14.5	-13.8	-13.0	-12.2	-11.5	-10.8	-10.1	-9.4	-8.6	
p	109.0	108.2	107.5	106.8	106.0	105.2	104.5	103.8	103.1	102.4	101.6	
h_{0i}	1".09	.76	.70	.76	1.02	1.31	1.45	1.39	1.23	1.03	.85	
	1".05	.85	.73	.73	1.10	1.46	1.51	1.43	1.36	.99	.84	
	1".10	.91	.68	.75	1.09	1.50	1.57	1.48	1.35	1.04	.86	
	1".16	.91	.72	.75	1.15	1.46	1.62	1.53	1.29	1.22	.87	
	1".14	.83		.70	1.11			1.55	1.37	1.14		
h_0	1".12	.86	.72	.75	1.10	1.44	1.55	1.48	1.32	1.08	.86	
h	+ ".05	- .15	- .25	- .17	+ .22	+ .62	+ .75	+ .72	+ .59	+ .38	+ .18	
h_1	- ".05	+ .19	+ .10	+ .08	+ .02	- .13	- .25	- .01	+ .06	+ .23	+ .18	
x	+ 4.2	.4	.6	.8	1.0	1.2	1.4	1.6	1.8	2.0	2.2	2.4
θ	+ 16.7	1.4	2.2	2.9	3.6	4.3		5.7	6.4	7.2	7.9	8.6
p	92.3	91.6	90.8	90.1	89.4	88.7		87.3	86.6	85.8	85.1	84.4
h_{0i}	".60	.34	.30	.36	.20	.08		.38	.48	.48	.90	1.12
	".48							.38	.43	.57	.88	1.19
	".47								.39	.55	.95	1.26
	".59									.64	.93	1.23
												1.23
h_0	".54	.34	.30	.36	.20	.08		.38	.43	.56	.92	1.21
h	- ".01	- .22	- .27	- .22	- .40	- .53		- .27	- .25	- .15	+ .17	+ .44
h_1	- ".04	- .12	- .04	+ .07	- .04	- .12		+ .13	- .04	- .20	- .02	+ .10

expedition.

contact.

— 2.2	— 2.0	— 1.8	— 1.6	— 1.4	— 1.2	— 1.0	— .8	— .6	— .4	— .2	.0	
— 7.9	— 7.2	— 6.4	— 5.7	— 5.0	— 4.3	— 3.6	— 2.9	— 2.2	— 1.4	— .7	.0	
100.9	100.2	99.4	98.7	98.0	97.3	96.6	95.9	95.2	94.4	93.7	93.0	
.72	.75	.77	.79	.80	.68	.56	.60	.73	.64	.60	.63	
.71	.71	.75	.82	.78	.61	.52	.62	.68	.68	.58	.67	
.69	.74	.74	.78	.81	.72	.49	.61	.75	.56	.64	.72	
.70	.75	.66	.87	.71	.58	.48	.58	.77	.69	.53	.69	
.71		.86			.68		.61					
.71	.74	.76	.82	.78	.65	.51	.60	.73	.64	.59	.68	
+ .07	+ .12	+ .16	+ .22	+ .21	+ .09	— .04	+ .05	+ .19	+ .10	+ .05	+ .14	
+ .12	+ .59	+ .36	+ .12	+ .21	+ .26	+ .11	+ .05	+ .04	— .11	— .14	+ .04	
2.6	2.8	3.0	3.2	3.4	3.6	3.8	4.0	4.2	4.4	4.6	4.8	5.0
9.4	10.1	10.8	11.5	12.2	13.0	13.8	14.5	15.2	16.0	16.8	17.5	18.2
83.6	82.9	82.2	81.5	80.8	80.0	79.2	78.5	77.8	77.0	76.2	75.5	74.8
1.13	1.06	1.16	1.14	1.00	.98	1.18	1.13	1.05	1.01	.87	1.07	1.33
1.18	1.07	1.19	1.11	.98	.98	1.19	1.09	1.06	.99	.88	1.15	1.29
1.26	1.02	1.21	1.11	.97	1.10	1.17	1.10	1.00	.96	.87	1.13	1.32
1.26	1.04	1.17	1.25	.95	1.00	1.23	1.10	.94	.91	.87	1.11	1.36
	1.04		1.19			1.17						
1.21	1.05	1.18	1.17	.99	1.03	1.20	1.12	1.02	.98	.88	1.13	1.33
+ .39	+ .20	+ .29	+ .23	+ .01	.00	+ .11	— .02	— .17	— .27	— .44	— .25	— .10
— .02	— .19	.00	+ .13	— .04	— .40	— .23	— .17	— .11	— .07	— .26	— .30	— .28

Table 14 (cont.).

Third

x	-7.6	-7.4	-7.2	-7.0	-6.8	-6.0	-5.8	-5.6	-5.4	-5.2	-5.0	-4.8	-4.6	-4.4	-4.2	-4.0	-3.8
θ	-28.5	-27.7	-26.9	-26.1	-25.3	-22.1	-21.3	-20.6	-19.8	-19.0	-18.3	-17.5	-16.7	-16.0	-15.3	-14.5	-13.8
p	245.3	246.1	246.9	247.7	248.5	251.7	252.5	253.2	254.0	254.8	255.5	256.3	257.1	257.8	258.5	259.3	260.0
h_{0i}	1".97	1.89	1.90	2.12	2.16	2.20	2.13	2.12	2.14	1.88	1.59	1.59	1.67	1.60	1.79	1.94	1.89
	2".02	1.91	1.98	2.17	2.22	2.27	2.19	2.18	2.15	1.90	1.55	1.72	1.66	1.61	1.82	1.96	1.85
	2".04	1.95	1.98	2.21	2.27	2.30	2.24	2.25	2.26	1.93	1.57	1.71	1.79	1.63	1.88	2.01	1.94
	2".00	1.95	1.97	2.20	2.30	2.33	2.25	2.25	2.28	2.09	1.66	1.65	1.77	1.64	1.86	1.95	1.90
							2.27	2.28	2.35	2.13					1.91	2.01	2.03
h_0	2".03	1.94	1.98	2.20	2.26	2.29	2.23	2.23	2.25	2.00	1.60	1.68	1.73	1.63	1.86	1.98	1.93
h	-1".04	-1.00	-.87	-.56	-.38	+.06	+.10	+.17	+.24	+.11	-.22	-.06	+.06	+.04	+.33	+.52	+.52
h_1	+ ".13	+.09	+.13	+.23	+.22	+.32	+.16	-.03	-.11	+.01	-.14	+.02	+.03	-.12	+.07	+.22	+.22

x	.0	.2	.4	.6	.8	1.0	1.2	1.4	1.6	1.8	2.0	2.2	2.4	2.6	2.8	3.0	3.2
θ	-0.0	-.7	+1.5	2.2	2.9	3.6	4.3	5.0	5.8	6.5	7.2	7.9	8.6	9.4	10.1	10.8	11.5
p	273.8	274.5	275.3	276.0	276.7	277.4	278.1	278.8	279.6	280.3	281.0	281.7	282.4	283.2	283.9	284.6	285.3
h_{0i}	1".22	1.31	1.52	1.60	.93	.95	1.11	.96	.65	.34	.34	.66	.90	.76	.70	.69	.67
	1".19	1.36	1.59	1.59	.96	.96	1.15	.90	.72	.37	.34	.63	.87	.74	.70	.66	.62
	1".26	1.42	1.59	1.67	1.05	.93	1.19	.98	.61	.49	.33	.74	.93	.74	.64	.66	.61
	1".28	1.39	1.66	1.71	.96	.98	1.20	1.00	.60		.33	.85	.91	.77	.75	.63	.68
		1.41			1.11	.97	1.25	1.12			.31	.74				.70	
											.34						
h_0	1".24	1.38	1.59	1.64	1.00	.96	1.18	.99	.64	.40	.33	.72	.90	.75	.70	.67	.64
h	+ ".51	+.67	+.89	+.94	+.31	+.27	+.49	+.29	-.06	-.31	-.39	-.02	+.15	-.02	-.09	-.15	-.20
h_1	+ ".18	-.03	-.19	+.19	-.09	+.04	+.24	.00	-.06	.00	-.10	+.08	+.20	+.36	+.31	+.23	+.18

Stockholm expedition.

contact.

-3.6	-3.4	-3.2	-3.0	-2.8	-2.6	-2.4	-2.2	-2.0	-1.8	-1.6	-1.4	-1.2	-1.0	-.8	-.6	-.4	-.2
-13.0	-12.3	-11.5	-10.8	-10.1	-9.3	-8.6	-7.9	-7.2	-6.4	-5.7	-5.0	-4.3	-3.6	-2.8	-2.1	-1.4	-.7
260.8	261.5	262.3	263.0	263.7	264.5	265.2	265.9	266.6	267.4	268.1	268.8	269.5	270.2	271.0	271.7	272.4	273.1
1.66	1.70	1.83	2.01	2.16	2.18	2.11	1.92	1.70	1.66	1.69	1.60	1.65	1.90	1.67	1.43	1.28	1.13
1.65	1.73	1.83	2.11	2.21	2.24	2.13	1.97	1.76	1.67	1.70	1.64	1.70	1.93	1.68	1.56	1.20	1.16
1.73	1.72	1.85	1.98	2.28	2.31	2.20	1.99	1.76	1.65	1.71	1.65	1.71	2.02	1.73	1.54	1.28	1.13
1.68	1.73	1.83	2.10	2.30	2.35	2.25	1.98	1.83	1.65	1.71	1.61	1.67	2.04	1.83	1.64	1.27	1.16
		1.86				2.29	2.00	1.78	1.70		1.60	1.66	2.07		1.56		
1.69	1.72	1.84	2.05	2.24	2.27	2.20	1.97	1.77	1.67	1.70	1.62	1.68	1.99	1.73	1.55	1.26	1.14
+ .35	+ .44	+ .62	+ .87	+ 1.11	+ 1.18	+ 1.15	+ .96	+ .81	+ .75	+ .81	+ .76	+ .84	+ 1.18	+ .94	+ .78	+ .51	+ .40
-.01	-.26	-.48	-.43	-.52	-.42	-.28	-.27	-.19	-.04	+ .06	-.80	-.66	-.07	-.09	-.06	-.08	+ .06
3.4	3.6	3.8	4.0	4.2	4.4	4.6	4.8	5.0	5.2	5.4	5.6	5.8	6.0	6.2	6.4	6.6	6.8
12.3	13.0	13.7	14.5	15.2	16.0	16.7	17.5	18.2	18.9	19.7	20.5	21.2	22.0	22.8	23.5	24.3	25.1
286.1	286.8	287.5	288.3	289.0	289.8	290.5	291.3	292.0	292.7	293.5	294.3	295.0	295.8	296.6	297.3	298.1	298.9
.56	.46	.35	.20	.36	.56	.50	.33	.47	.59	.82	.92	1.14	1.30	1.41	1.57	1.64	1.69
.61	.53	.39	.20		.47	.61	.33	.29	.53	.79	.92	1.06	1.34	1.44	1.56	1.66	1.69
.57	.48		.20		.44	.57	.32		.54	.82	.96	1.17	1.35	1.46	1.60	1.73	1.75
.55			.20		.48	.59	.31		.61	.80	.88	1.23	1.35	1.48	1.64	1.71	1.76
.58					.59	.30			.63	.86				1.46	1.62	1.79	1.79
						.35											
.57	.49	.37	.20	.37	.50	.58	.33	.39	.59	.83	.93	1.16	1.35	1.46	1.61	1.72	1.75
-.30	-.41	-.56	-.79	-.65	-.58	-.54	-.82	-.82	-.67	-.48	-.45	-.27	-.15	-.11	-.02	.00	-.04
+ .21	+ .18	+ .07	-.09	+ .15	+ .27	+ .31	+ .02	-.02	+ .07	+ .15	-.02	-.01	.00	-.08	-.01	+ .21	+ .25

CHAPTER VI

Summary and concluding remarks

Summary. The spectrophotometric procedure of timing a total solar eclipse utilizes the fact that the intensity at the extreme solar limb falls off very rapidly. When the Moon during the progress of the eclipse covers or uncovers the extreme photospheric layers, the light allowed to reach a terrestrial observer will accordingly decrease or increase with great rapidity. The moments of the inner contacts are settled from a study of the rate of change of light at both contacts.

A study of the distribution of energy in this region is rendered possible by using as recording instrument a parabolic mirror or lens of moderate focal length, in conjunction with an objective prism and a registering film camera. The flash spectra obtained thereby, which are timed accurately by means of a convenient timing device, are subjected to an objective photometry carried out in the continuous spectrum along a path at right angles to the direction of dispersion. The gradient of intensity is derived by studying the rate of change of intensity at a selected point in the continuous spectrum. The obtained gradient may be compared with the results arrived at on a purely theoretical basis. The gradients derived by LINDBLAD and the present author are in agreement with WILDT's theoretical considerations concerning the sharpness of the solar limb.

The empirical intensity gradient which holds all around the solar border is utilized to establish the relative position of the edges of the two celestial bodies. The solar edge, in this procedure, is defined to coincide with a certain isophote of stated intensity situated within the interval of photometered intensities. The connection between intensity and height above this solar isophote as condensed in the empirical gradient furnishes the momentary heights of any contour point at the moments of the successive flash spectrum exposures. The different heights thus obtained may, by accounting for the relative motion, be combined to give an accurate mean photometric height, valid for a certain moment. The shape and position of the lunar contour, as referred to the adopted isophote, is obtained by carrying through the procedure mentioned at a convenient number of points along the lunar edge. The result is two well-defined photometric contours, extending over some 50° in position angle at each contact. The moments of contact are achieved by introducing a mean lunar level considered as representing the lunar border for contact purposes.

So far the discussion has been about the absolute contact problem. In the geodetic, that is to say the relative, contact problem the matter is to state the difference between the contact times at two stations of observation. In this case, when no systematic errors interfere and when differential libration is disregarded, the problem is reduced to a study of the mutual agreement between the contours determined by the two observers. It is revealed from the present investigation that the accuracy of the photometric method as derived on the basis of this agreement permits to obtain results of geodetic interest. It is pointed out that a considerably greater accuracy might be obtained by employing instruments of equal focal length.

Concluding remarks. It is obvious that a purely photometric treatment of the contact problem is able to offer several advantages over the classical methods which consider the determination of contacts as a question of a more or less geometrical character. It is not necessary to repeat in detail these advantages which have been displayed on several occasions in the foregoing. We shall instead confine ourselves here to making a comparison of the present photometric procedure and that proposed by the Argentinian astronomers PLATZECK, MAIZTEGUI and GAVIOLA (p. 8). Moreover, some remarks will be given concerning a future application of the present photometric method.

The arrangement advanced by the Argentinian astronomers differs from the actual photometric method in that the registrations of the Sun's illumination bear upon the total amount of light emitted from the extreme photospheric layers. This fact makes it necessary to extend the measurements to concern a portion of several seconds of arc at the extreme solar limb since, at the outermost edge, the irregularities of the lunar border would distort seriously the symmetry of the light curves at the two contacts. Thus, the measurements do not become trustworthy until in a region where the gradient is less than at the very limb and, accordingly, the lightcurve a less sensible indicator of the relative position. Corrections for the lunar limb topography presuppose a rather accurate knowledge not only of the shape of this border but also of the true distribution of energy at the solar limb. It might from this point of view prove useful to combine the two photometric procedures; in doing so, the flash spectrum method would be able to provide the other with data concerning the true shape and orientation of the lunar marginal zone whereby both of them would be put on the same basis as regards the determination of contacts. The registration of total light, on the other hand, would widen considerably the photometric basis for an accurate timing of a total solar eclipse. It seems that an arrangement of this kind should be particularly well adapted for a successful carrying through of the geodetic project.

With reference to the present procedure, it has previously been pointed out that at future attempts the stations of observation should be located as close as possible to the central line of the total track. The reason for this is that, by so doing, the relative contact problem will reduce to a differential problem in proper sense because then the photometric contours will refer to the same portion of the lunar border and accordingly should agree very closely (apart from small departures due to differential libration). The experiences gained at the present attempt have revealed, moreover, that it is very important to use registering instruments of equal focal length, thereby causing the levelling of the photometric profiles (due to the small scales) to assume the same value. A reasonable standardization of other instrumental properties and the treatment of the photometric materials as well would, of course, only be in favour of the results deduced on such a basis. A somewhat shorter time of exposure at the cinematographic recording, for instance some few thousandths of a second, may be realized by adjusting the effective focal ratio to a larger value. Certain advantages for the contact problem might be achieved by extending, in one way or other, the interval of intensity covered by the present photometric registration.

The appearance of the lunar marginal zones will differ to a certain extent at the two ends of a long geodetical basis. It is important to be aware of this effect of differential libration for the sake of the geodetic project. It seems as if the work of WATTS and ADAMS should be particularly valuable for this purpose.

List of references.

- [1] AUWERS, A., Der Sonnendurchmesser und der Venusdurchmesser nach den Beobachtungen an den Heliometern der deutschen Venus-Expeditionen. AN 128, 361, 1891.
- [2] BANACHIEWICZ, T., Polnische Sonnenfinsternisexpedition 1927. Comptes Rendus de la 4. Séance de la Com Géod Balt, pp 49, 161, Helsinki 1929.
- [3] —, Expéditions polonaises pour l'observation de l'éclipse du Soleil, 1936 Juin 19. Acta Astronomica, Série c, Vol 3, p 16, 1936.
- [4] BONSDORFF, I., Die astronomisch-geodätischen Arbeiten während der Sonnenfinsternis den 9. Juli 1945. Tätigkeit der Balt Geod Kommission in den Jahren 1942—43, p 12, 1943.
- [5] GAVIOLA, E., On seeing, fine structure of stellar images, and inversion layer spectra. AJ 54, p 155, 1949.
- [6] GRÖNSTRAND, H. O., The total solar eclipse of 1945, July 9. Prediction for Scandinavia. Stockholm Ann 14, No. 7, 1944.
- [7] —, The total solar eclipse of 1945, July 9. Prediction for Finland. Åbo Akad Obs Medd 3, 1944.
- [8] —, The total eclipse of the Sun on June 30, 1954. Prediction for Scandinavia. Stockholm Ann 16, No. 2, 1950.
- [9] HAMAKER, H. C., Reflectivity and emissivity of tungsten, diss., Utrecht-Amsterdam 1934.
- [10] HAYN, F., Selenographische Koordinaten, III und IV Abh. Band 30 bzw 33 der Abh der math-phys Klasse der Königl Sächs Ges der Wissensch, Leipzig 1907 bzw 1914.
- [11] —, Die Gestalt der Sonne. AN 220, 113, 1924.
- [12] —, Beobachtungen der Sonnenfinsternis 1927 Juni 29 auf der Sternwarte in Leipzig. AN 233, 183, 1928.
- [13] JASNOŹEWSKI, J., The sending of the 1936 June 19 solar eclipse timesignals. Acta Astronomica, Série c, Vol 3, p 38, 1936.
- [14] JULIUS, W. H., A new method for determining the rate of decrease of the radiative power from the center toward the limb of the solar disc. ApJ 23, p 312, 1906.
- [15] KINNEY, W., Operation eclipse: 1948. The National Geographic Magazine, Vol XCV, p 325, 1949.
- [16] KORDYLEWSKI, K., Die polnische Sonnenfinsternis-Expedition nach Schwedisch-Lappland zur totalen Finsternis, 1927 Juni 29. Acta Astronomica, Série b, Vol 1, p 133, 1932.
- [17] KÜHL, A., Die Reduktion von Fernrohrbeobachtungen wegen Kontrastfehlers. Probleme der Astronomie (Festschrift für Hugo v. Seeliger), p 372, Berlin 1924.
- [18] LINDBLAD, B., Solens flashspektrum vid den totala solförmörkelsen den 29 juni 1927. Kosmos 1929, Band 7, p 163, Stockholm 1929.
- [19] —, Bemerkungen über die Messung der Kontaktzeiten während der totalen Sonnenfinsternis den 9. Juli 1945. Tätigkeit der Balt Geod Kommission in den Jahren 1942—43, p 17, 1943.
- [20] —, Stockholms Observatoriums expeditioner för observation av den totala solförmörkelsen den 9 juli 1945. Populär Astronomisk Tidskrift XXVI, p 81, Stockholm 1945.
- [21] MALMQUIST, K. G., H. NORINDER and W. STOFFREGEN, On the registration of the exact time of each exposure by cinematographic photography. Arkiv för Mat Astr och Fysik, Band 33 B, No. 3, 1945.

- [22] MALMQUIST, K. G., The Uppsala Observatory expeditions to North Sweden for studying the total solar eclipse of July 9, 1945. Kungl. Svenska Vetenskapsakademiens Handlingar, Fjärde Serien. Band 1, No. 3, 1949.
- [23] MINNAERT, M., The distribution of energy near the limb of the Sun. MN 89, p 197, 1928.
- [24] MINNAERT, M. and J. HOUTGAST, Photometrische Untersuchungen über das Funkeln der Fixsterne. ZfA 10, p 86, 1935.
- [24^a] O'KEEFE, J. A., A new determination of the lunar parallax. AJ 55, p 177, 1950.
- [25] PANAY, T. N., Note on measurements of the films of the solar eclipse, May 9-8, 1948. Transactions American Geophysical Union, Vol 31, No. 6, 1950.
- [26] PLATZECK, R., A. MAIZTEGUI and E. GAVIOLA, Non-focal methods for determining 2nd and 3rd contacts in total and center in annular solar eclipses. Publication dedicated to Ilmari Bonsdorff on the occasion of his 70th anniversary, p 191, Helsinki 1949.
- [27] RICHARDSON, R. A. and E. O. HULBURT, The illumination from the solar crescent near totality of the eclipse of May 20, 1947. ApJ 111, p 99, 1950.
- [28] RIGHINI, G., Profondità ottica e pressione della cromosfera solare in relazione alla percentuale di idrogeno. Memorie della Società Astronomica Italiana, Nuova Serie 15, p 217, 1943.
- [29] SCHLESINGER, F., Some aspects of astronomical photography of precision. MN 87, p 506, 1927.
- [30] SCHWARZSCHILD, K., Über die totale Sonnenfinsternis vom 30. August 1905. Astr Mitt der Königl Sternwarte zu Göttingen, 13. Teil, 1906.
- [31] SUNDMAN, K. F., The motions of the Moon and the Sun at the solar eclipse of 1945, July 9th. Extrait de "l'Activité de la Com Géod Balt pendant les années 1944-47", p 63, Helsinki 1948.
- [32] UNSÖLD, A., Physik der Sternatmosphären, p 394. Berlin 1938.
- [33] WATTS, C. B. and A. N. ADAMS, Photographic and photoelectric technique for mapping the marginal zone of the Moon. AJ 55, p 81, 1950. Cf. also Sky and Telescope, Vol IX, p 134, 1950.
- [34] WILDT, R., An interpretation of the heights of lines in the solar chromosphere. ApJ 105, p 36, 1947.
- [35] ÖHMAN, Y., On some applications of the flicker method in photoelectric measurements. Arkiv för Mat Astr och Fysik, Band 29 B, No. 12; Stockholm Medd, No. 54, 1943.
- [36] Proceeding of the Greenwich Observatory, MN 109, p 155, 1949.
- [37] Proceedings of the Finnish Academy of Science and Letters 1948, p 99, Helsinki 1950. With contributions by I. Bonsdorff, T. J. Kukkamäki and R. A. Hirvonen.
- [38] The Swedish solar eclipse expeditions to West-Africa and Brazil in 1947. Rikets allmänna kartverk, Meddelande No. 11 (Stockholm 1951). With contributions by B. Aurell, E. Bodén and G. Larsson-Leander, A. Leijonhufvud, E. Bergstrand, Y. Öhman, J. Lindgren, L. Stigmark, T. Elvius, J. Ramberg, N. Hansson and H. Kristenson, Å. Wallenquist.

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